How Much Better is Commitment Policy Than Discretionary Policy? Evidence from Six Developed Economies

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January 25, 2020

Abstract

Much has been written on how an active central bank produces inflation outcomes above and beyond what commitment policy would produce. This paper contributes to this body of literature by simulating from the state estimates of both commitment and discretionary policy equilibria in a familiar dynamic New-Keynesian framework. Optimal interest rate and inflation rate policies are derived under the two regimes for six developed economies. The model is estimated using Bayesian methods employing a random-walk Metropolis-Hastings algorithm. Optimal inflation and interest rate policies for each of the economies are simulated. Results suggest that the simulated inflation induced by discretionary policy is not significantly different from commitment policy after 2000 for five of the six countries (including the U.S.). Simulated commitment interest rate policy is on average 1.9% higher at the center of the distribution, suggesting that discretionary interest rate policy is on average more often loose compared to commitment interest rate policy. Simulations of the average inflation deviation and welfare loss of discretion policy indicate are greatest when the central bank exhibits low preference for inflation targeting and high preference for output stability.

JEL Classification: E31, E52, E58, E61

Keywords: Optimal monetary policy, timeless perspective policy, discretionary policy, monetary policy deviations, Bayesian inference

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1 Introduction

The inconsistency of optimal monetary policy is a phenomenon that has been well studied in the theoretical monetary policy literature. Kydland and Prescott (1977) discuss how the central bank, when it has discretionary power, can seek to stabilize prices, and the outcome will result in an equilibrium that is optimal but inconsistent. Put another way, the central bank in an effort to target an unemployment rate below the natural rate induces a level of inflation above that which society would prefer. Barro and Gordon (1983) expand further, suggesting that a monetary authority seeking to surprise the public induces an inflation bias. However, in this model society recognizes that the monetary authority has an incentive to deceive them, so the inflation bias is an average level of inflation above how private sector agents formulate expectations. This definition implies that either discretionary or commitment policy rules could in theory produce an inflation bias. The notion discussed by Kydland and Prescott (1977) is somewhat different describing the difference between discretionary and commitment policy outcomes in a control theory framework[1].

Much of the recent empirical literature analyzing inflation outcomes models monetary policy preferences as asymmetric in functional form. This so-called asymmetric preferences literature relaxes the a priori assumption that the policy maker responds with an equal sense of urgency during expansionary and recessionary phases. This notion first appears in the literature by Cukierman and Gerlach (2003), Nobay and Peel (2003), and Ruge-Murcia (2003), where it is shown that policy makers targeting natural output or natural unemployment can induce an inflationary bias by their own response asymmetry. Cukierman and Gerlach (2003) shows that even if the central bank targets the nominal level of employment, a bias is produced due to the uncertainty around economic conditions. It should however be noted that this type of bias is not strictly due to the time inconsistency of optimal decisions, but also because the policy maker's preferences are warped.

Surico (2007) employing the aforementioned asymmetric preferences framework estimated an

[1]For the remainder of this paper the term 'inflation bias' is used more to describe the difference in policy rule outcomes and not in the classic Barro and Gordon (1983) sense.

average inflation bias of approximately 1.5% for the U.S over the period corresponding to the great moderation[2]. Anderson, Kim and Yun (2010) employ a projection method approach in order to examine discretionary and commitment policy. They show that for plausible values of the model parameters the difference between discretionary and commitment policy should be somewhere between 1% to 6%. Billi (2011) in a calibrated model shows that when the central bank commits at least to an inertial Taylor rule the inflation bias is eliminated[3]. These empirical analyses however are effectively measuring an average inflation bias. This method may not be appropriate if the inflation bias is non-stationary over time. Additionally, it tells us little about how the bias might change when the fundamentals of the economy vary.

The monetary policy actions taken by central banks around the world in response to the great recession raise important questions about measuring the observed effects of commitment versus discretionary policy. Since optimal commitment policy does not necessarily involve zero inflation and zero output gap (Kirsanova et al.,2009), the inflation bias is not just discretionary policy inflation rules above and beyond zero inflation. Additionally, commitment policy inflation is not necessarily zero inflation for all periods. If the central bank's policy targets are not assumed to be constant over time, commitment policy inflation outcomes inherit dynamics that are statistically different from zero more often than not. When does discretionary policy yield different inflation outcomes for society than what commitment policy would prescribe? In low inflation times, do these rules vary more than in high inflation times? Finally, as a central bank alters its stance on defending inflation relative to output gap stabilization, how does this effect the average difference between these two policy regimes?

This paper contributes to the literature on optimal monetary policy in some unique ways. First, commitment (timeless perspective) and discretionary policy interest rate and inflation rate rules are solved for in a dynamic equilibrium model with a time-varying inflation and interest rate target.

[2] Approximately mid 1980's to mid 2000's.

[3] Its important to note however that both Anderson, Kim and Yun (2010) and Billi (2011) are strictly numerical and not employing observed data.

Since these targets are not constant over time,[4] it is shown that relaxing this assumption produces additional dynamics in the optimal policy rules and not just imposed ad hoc. These dynamics are more pronounced under a strategy of commitment rather than discretion. The commitment interest rate policy rule nests the discretionary interest rate policy rule as special case. Second, this model is confronted with data for six developed economies; Australia, Canada, the European Union (Euro zone countries), Japan, the United Kingdom, and the United States. It is estimated using Bayesian methods employing a random-walk Metropolis-Hastings MCMC algorithm. Joint posterior density estimates are provided for all countries. Third, the posterior distribution of the parameters as well as estimates from the state distribution are used to simulate and produce estimates of both commitment and discretionary policy over time. Since the joint posterior density is used, the simulations produce a distribution of possible inflation and interest rate paths that can be interpreted similar to confidence bands from impulse response functions. This provides a more accurate picture of how policy could hypothetically evolve under the different regimes. Finally, the empirical data and the posterior densities are used to simulate both average inflation deviations and welfare loss of discretion policy relative to commitment policy for different possible combinations of inflation versus output gap targeting behavior.

Estimations suggest some important results. First, there are multiple periods of observable significant deviations between the two inflation policies prior to 2000 for all countries except the United Kingdom. Post 2000, this deviation disappears for all countries except Australia. The deviation decreases after 2000 at the mean by as little as 0.5% (Canada) and as much as 1.5% (U.S.). The average inflation policy deviation for all six economies at the mean is about 1.39% over the full sample period. Second, interest rate policy simulations imply an interest rate policy deviation as well. Discretionary interest rate policy is on average lower than commitment policy implying that discretionary policy is more often loose than tight (evidence of asymmetric preferences). This

[4] See Dossche and Everaert (2005), Ireland (2007), Leigh (2008), and Scott (2016) for a more thorough analysis of this hypothesis.

interest rate bias largely mirrors the inflation policy deviations due to the underlying Fisher effect. The interest rate bias is present for all countries except the U.K. prior to 2000. Australia's interest rate deviation is still present after 2000. Japan's interest rate deviation disappears during the mid 1990's (the beginning of its low growth phase). Canada's interest rate deviation disappears during the period corresponding to the global financial crisis. Third, simulations for the average inflation policy deviations for differing degrees of relative output gap and inflation targeting behavior indicate that higher degrees of output gap targeting combined with lower aversion to inflation leads to a higher potential inflation bias. For four out of six economies the potential bias is greater than the unconditional average for observed inflation. The lowest levels of potential bias correspond to low degrees of output gap targeting and higher degrees of inflation targeting behavior. Fourth, simulations at the posterior mean show that for most of the economies high degrees of output gap preferences and low inflation sensitivity produces the highest discretionary welfare loss relative to commitment. The lowest degree of discretionary relative welfare loss occurs when the monetary authority places little weight on output gap movements. Relative welfare loss over time varies considerably within the full sample period and is driven by both the divergence of discretionary and commitment policy credibility intervals as well as the variance of those intervals.

The remainder of this paper is organized as follows. Section 2 details the model and solution under both discretion and commitment. Section 3 explains the estimation procedure and discusses the data used. Section 4 summarizes the findings of the empirical estimation and simulation. Section 5 concludes with a brief discussion.

2 Model

The economy is assumed to evolve according to the canonical form of a standard log-linearized dynamic New-Keynesian model. The consumer's forward-looking Euler equation is given by (1). Here x_t is the deviation in output under sticky prices, (y_t), from output under flexible prices, (y_t^n)

or $x_t \equiv y_t - y_t^n$. The expectations operator, E_t, denotes conditional expectations formed using all available information at time t. σ represents the inverse of the inter-temporal elasticity of substitution which summarizes the rate of substitution between consumption today versus consumption tomorrow. The observed interest rate is given by i_t and the observed inflation rate is defined by π_t. ϵ_t^d is an error term that is assumed orthogonal to any rational expectations errors and follows $\epsilon_t^d \sim (0, \sigma_d^2)$. As a whole Equation (1) summarizes the demand side of the economy.

$$x_t = E_t x_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) + \epsilon_t^d \tag{1}$$

The supply side of the economy is governed by a forward-looking New-Keynesian Phillips curve (NKPC), Equation (2). As in Equation (1) π_t, x_t, and E_t are the inflation rate, output gap, and conditional expectations operator respectively. β is the discount factor of the consumer's utility maximization problem. κ is a reduced form parameter that is a function of the Calvo (1983) price adjustment lottery. Finally, ϵ_t^s represents a cost-push inflationary shock and once again is orthogonal to any rational expectations errors and evolves according to $\epsilon_t^s \sim (0, \sigma_s^2)$.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \epsilon_t^s \tag{2}$$

No assumptions, such as serial correlation, are imposed on the shock terms ϵ_t^d or ϵ_t^s since they have no bearing the policy equations or empirical exercise as will be shown below.

Both commitment and discretionary policy are derived from an optimizing agent framework. The monetary authority faces preferences that are quadratic in nature derived by (3).

$$L_t = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\pi_t - \pi_t^*\right)^2 + \frac{\phi}{2} x_t^2 + \frac{\delta}{2}\left(i_t - i_t^*\right)^2\right] \tag{3}$$

The central bank, which is assumed to act optimally, seeks to minimize societal loss, L_t which is the conditional expectational sum of future losses. Deviations of inflation from a time varying inflation target (π_t^*) are normalized to one. ϕ and δ are the relative aversion parameters to deviations in output from natural output and the interest rate from an implicit target (i_t^*) respectively. Larger values for these parameters imply a stronger aversion (and thus a higher degree of associated loss) to deviations.

$$\pi_t^* = \rho\pi_{t-1}^* + \epsilon_t^\pi, \tag{4}$$

The inflation target is assumed to follow a first-order autoregressive process according to Equation (4). This is a similar formulation to Dossche and Everaert (2005), Leigh (2008) and Scott (2016).

$$i_t^* = \bar{r} + \pi_t^*. \tag{5}$$

The interest rate target is assumed to follow the same underlying dynamics as the inflation target since they are linked according to the Fisher equation. The interest rate target is assumed to follow the inflation target according to (5)[5].

2.1 Optimal Commitment Policy

Optimal commitment policy suffers from an initial condition problem. For policy makers to commit to a policy in the past there must necessarily be a point of reference where the initial conditions of the economy are specified. To circumvent this problem commitment policy can be defined according to Woodford et al. (1999)'s timeless perspective policy (the policy that the central bank wished it had committed to at some point in the distant past). Using this definition, optimal commitment policy can be found by minimizing L_t, Equation 3, w.r.t $\{\pi_t,\ y_t,\ i_t\}_0^\infty$ subject to (1) and (2). The generalized system of linear first-order necessary conditions (FOC) for this problem are characterized

[5]Another specification for the inflation target might also look like $\pi_t^* = \alpha + \rho\pi_{t-1}^* + \epsilon_t^\pi$. Under this specification however α and $\bar{r}$ would not be independently identifiable in the empirical estimation. Thus by assuming that the inflation target follows an AR(1) with no drift, identification of $\bar{r}$ is possible, otherwise the constant in Equation 19 would be a nuisance parameter.

by Equations (6) - (10).

$$
\begin{aligned}
(\pi_t - \pi_t^*) - \lambda_{\pi t} &= 0, \quad t = 1 && (6)\\
\phi x_t + \kappa \lambda_{\pi t} - \lambda_{xt} &= 0, \quad t = 1 && (7)\\
(\pi_t - \pi_t^*) + \lambda_{\pi t-1} + \frac{\sigma}{\beta}\lambda_{xt-1} - \lambda_{\pi t} &= 0, \quad t = 2, 3, \ldots && (8)\\
\phi x_t - \frac{1}{\beta}\lambda_{xt-1} + \kappa\lambda_{\pi t} - \lambda_{xt} &= 0, \quad t = 2, 3, \ldots && (9)\\
\delta\left(i_t - i_t^*\right) - \sigma\lambda_{xt} &= 0, \quad t = 1, 2, 3, \ldots && (10)
\end{aligned}
$$

These FOC's define the time inconsistency problem of the central bank. Commitment policy arises from Equations (8), (9) and (10) while discretionary policy arises from (6), (7) and (10). Using Equations (8) - (10) and the law of iterated expectations, we can define the FOC for optimal commitment interest rate policy as Equation 11.

$$
\begin{aligned}
0 \;=\; & (\pi_t - \pi_t^*) + \frac{\delta}{\sigma\kappa}\left(i_{t-1} - i_{t-1}^*\right) + \frac{\delta}{\sigma\beta\kappa}\left(i_{t-2} - i_{t-2}^*\right) - \frac{\phi}{\kappa}x_{t-1}\\
+ \; & \frac{\delta}{\beta}\left(i_{t-1} - i_{t-1}^*\right) - \frac{\delta}{\sigma\kappa}\left(i_t - i_t^*\right) - \frac{\delta}{\sigma\beta\kappa}\left(i_{t-1} - i_{t-1}^*\right) + \frac{\phi}{\kappa}x_t \qquad (11)
\end{aligned}
$$

(11) reduces after some small algebra to (12).

$$
\Delta i_t = \frac{\sigma\kappa}{\delta}\{\pi_t - \pi_t^*\} + \Delta i_t^* + \frac{\sigma\phi}{\delta}\Delta x_t + \frac{\sigma\kappa}{\beta}\{i_{t-1} - i_{t-1}^*\} - \frac{1}{\beta}\{\Delta i_{t-1} - \Delta i_{t-1}^*\} + \epsilon_t^c \qquad (12)
$$

This interest rate policy rule is similar to Dennis (2010) in form, but it implies more persistence since the inflation and interest rate target are not assumed constant. Commitment policy also implies persistence in the output gap, this is the so called speed limit to monetary policy first discussed by Walsh (2003).

The FOC's contained in (6) - (10) also contain an optimal path for inflation under both discretion and timeless perspective commitment policy. Optimal inflation under a strategy of timeless perspective commitment is given by Equation 8. After some similar algebra Equation 8 yields (13).

$$\pi_t = \pi_t^* - \frac{\phi}{\kappa}\Delta x_t + \frac{\delta}{\sigma\kappa}\left[\Delta i_t - \Delta i_t^*\right] + \frac{\delta}{\sigma\beta\kappa}\left[\Delta i_{t-1} - \Delta i_{t-1}^*\right] - \frac{\delta}{\beta}\left(i_{t-1} - i_{t-1}^*\right) \tag{13}$$

This optimal rule for inflation captures much of the same underlying persistence as the interest rate rule in Equation 12.

2.2 Optimal Discretionary Policy

Discretionary policy does not suffer from the same initial condition problem as commitment policy. Here every period is an initial condition by which policy makers respond. Discretionary policy can then be found by substituting (1) and (2) into the static form of (3) at $t = 1$ which results in (14).

$$\begin{aligned} L_t \quad = \quad & E_t \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{1}{2}\right) \left(\beta E_t \pi_{t+1} + \kappa \left(E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1}\right) + \varepsilon_t^d\right) + \varepsilon_t^s - \pi_t^*\right)^2 \right. \\ + \quad & \left. \frac{\phi}{2}\left(E_t x_{t+1} - \sigma\left(i_t - E_t\pi_{t+1}\right) + \varepsilon_t^d\right)^2 + \frac{\delta}{2}\left(i_t - i_t^*\right)^2 \right] \end{aligned} \tag{14}$$

Minimizing Equation 14 w.r.t. i_t implies the following FOC[6].

$$-\sigma\left\{\kappa\left[\left(\pi_t - \pi_t^*\right)\right] + \phi x_t\right\} + \delta\left(i_t - i_t^*\right) = 0 \tag{15}$$

or

$$\Delta i_t = \frac{\sigma\kappa}{\delta}\{\pi_t - \pi_t^*\} + \frac{\sigma\phi}{\delta}x_t + i_t^* - i_{t-1} + \epsilon_t^d \tag{16}$$

Note here that the policy rule under discretion is not characterized in levels but in differences in order to show that optimal discretionary interest rate policy is nested within optimal commitment interest rate policy. The backward looking dynamics are no longer present under a discretionary equilibrium, highlighting once again the dynamic tradeoff over time that the central bank faces.

[6] One can also use (6), (7), and (10) to get to (15).

Optimal inflation under discretion is found similar to the commitment case, but here it is given by (6). After some similar algebra we get

$$\pi_t = \pi_t^* - \frac{\phi}{\kappa}x_t + \frac{\delta}{\sigma\kappa}\{i_t - i_t^*\} + \frac{\delta}{\sigma\beta\kappa}\{i_{t-1} - i_{t-1}^*\} \tag{17}$$

Here also, optimal inflation under discretion implies less persistence than optimal inflation under commitment.

3 Data and Method

The empirical estimation of this model will center around the joint estimation of the interest rate policy rule and the unobserved inflation target dynamics. Since the interest rate target dynamics are a scaled form of the inflation target dynamics, this can be iteratively substituted out of Equation (12) which simplifies to (18).

$$\Delta i_t = \frac{\sigma\kappa}{\beta}\bar{r} + \frac{\sigma\kappa}{\delta}\{\pi_t - \pi_t^*\} + \Delta\pi_t^* + \frac{\sigma\phi}{\delta}\Delta x_t + \frac{\sigma\kappa}{\beta}\{i_{t-1} - \pi_{t-1}^*\} - \frac{1}{\beta}\{\Delta i_{t-1} - \Delta\pi_{t-1}^*\} + \epsilon_t^c \tag{18}$$

A reduced form version of this commitment policy rule is then represented by,

$$\Delta i_t = c\bar{r} + a\{\pi_t - \pi_t^*\} + \Delta\pi_t^* + b\Delta x_t + c\{i_{t-1} - \pi_{t-1}^*\} - d\{\Delta i_{t-1} - \Delta\pi_{t-1}^*\} + \epsilon_t^c, \tag{19}$$

where $a = \frac{\sigma\kappa}{\delta}$, $b = \frac{\sigma\phi}{\delta}$, $c = \frac{\sigma\kappa}{\beta}$, and $d = \frac{1}{\beta}$. Here the timeless perspective policy rule is measured in differences. It contains a few familiar components such as an inflationary gap and an output gap, but also contains a lag in the output gap and some additional persistence beyond a simple non-dynamic Taylor (1993) rule.

The discretion policy model can be found similarly. Substituting (5) into (16) produces (20).

$$\Delta i_t = \bar{r} + \frac{\sigma\kappa}{\delta}\{\pi_t - \pi_t^*\} + \frac{\sigma\phi}{\delta}x_t + \pi_t^* - i_{t-1} + \epsilon_t^d \tag{20}$$

This can likewise be expressed in a reduced form as (21).

$$\Delta i_t = \bar{r} + a\{\pi_t - \pi_t^*\} + bx_t + \pi_t^* - i_{t-1} + \epsilon_t^d \tag{21}$$

The reduced form parameters a and b are defined as in (19). Discretionary policy necessarily implies a greater sensitivity to the ever changing initial conditions for the economy, thus the policy rule imparts less persistence on the interest rate. Additionally, it is worth noting that the discretionary policy rule (both reduced and structural) is nested in the commitment policy rule. Thus discretionary policy is a special case of commitment policy.

Using the definitions from (13) along with (5), optimal inflation under timeless perspective policy in a reduced form is given by (22).

$$\pi_t^c = \frac{c}{a}\bar{r} + \pi_t^* - \frac{b}{a}\Delta x_t + \frac{1}{a}\left[\Delta i_t - \Delta\pi_t^*\right] + \frac{d}{a}\left[\Delta i_{t-1} - \Delta\pi_{t-1}^*\right] - \frac{c}{a}\left(i_{t-1} - \pi_{t-1}^*\right) \tag{22}$$

Here the definitions for a, b, c, and d are the same as in (19) and the empirical estimation. Likewise under discretion, substituting (5) into (17) produces in its reduced form

$$\pi_t^d = \bar{r}\frac{1}{a}(d-1) + \pi_t^* - \frac{b}{a}x_t + \frac{1}{a}\{i_t - \pi_t^*\} + \frac{d}{a}\{i_{t-1} - \pi_{t-1}^*\}. \tag{23}$$

Armed with these optimal policy rules, the average inflation policy deviation is given by Equation (24).

$$\bar{\pi}_t^{deviation} = \sum_{t=1}^{T-1}\bar{r}\frac{d-c-1}{a} - \frac{b}{a}x_{t-1} + \frac{1-c}{a}\{i_{t-1} - \pi_{t-1}^*\} - \frac{d}{a}\{i_{t-2} - \pi_{t-2}^*\} \tag{24}$$

The time dynamics that comprise the difference in optimal inflation outcomes summarizes the degree to which the monetary authority must look beyond the initial conditions of the economy. These dynamics also highlight the inertia that is placed on optimal policy making. Separately, estimates for the reduced form coefficients a and b quantify relative inflation and output gap targeting behavior respectively. This can also be seen in Equation 24 where the coefficient on the output gap lag is $-\frac{b}{a}$. The degree of sensitivity of the average inflation deviation as well as relative welfare loss from discretionary policy to changes in these parameters is examined below.

3.1 Estimation strategy

Equations (19) and (4) represent the system of processes to be estimated. Together these equations can be cast into state-space form. Following Hamilton (1994) notation, π_t^* and ϵ_t^π are unobserved components whose evolution through time comprises the state, Equation 25.

$$\xi_t = F\xi_{t-1} + Qv_t \tag{25}$$

The column vector, $\xi_{t|t-1}$ is modeled to contain the unobserved components of the series where

$$\xi_t = \begin{bmatrix} \pi_t^* \\ \pi_{t-1}^* \\ \pi_{t-2}^* \\ \epsilon_t^\pi \end{bmatrix}, \quad F = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad v_t = [\epsilon_t^\pi]$$

The first difference of the interest rate, Δi_t, defines the observation equation which is related to the unobserved state through theory via the coefficients in (19). The observation equation is given by Equation 26

$$Y_t = AX_t + H\xi_t + Wr_t \tag{26}$$

where

$$Y_t = [\Delta i_t], \quad A = \left[\begin{array}{ccccc} c\bar{r} & (1-c)\,a & (1-c)\,b & c & -(1-c)\,d \end{array} \right], \quad X_t = \left[\begin{array}{c} 1 \\ \pi_t \\ \Delta x_t \\ i_{t-1} \\ \Delta i_{t-1} \end{array} \right]$$

$$H = \left[\begin{array}{cccc} (1-c)\,(1-a) & ((1-c)\,d-1) & -(1-c)\,d & 0 \end{array} \right], \quad W = [1], \quad r_t = [\epsilon_t^c].$$

This type of formulation is very similar in nature to the model estimated in Leigh (2008). It is worth noting that many possible state-space formulations may exist, but they all yield the same predictions of the state so long as they span the same subspace of the parameters (Hamilton (1994)). The log likelihood function for the system is directly observable by

$$ll = \sum_{t=1}^{T} \ln \left[(2\pi)^{-\frac{1}{2}} \det \left(H' P_{t|t-1} H + R \right)^{-\frac{1}{2}} \exp \left(-0.5 e'_{t|t-1} \left(H' P_{t|t-1} H + R \right)^{-1} e_{t|t-1} \right) \right],$$

where $P_{t|t-1} \equiv Q v_t Q'$ and $e_{t|t-1} \equiv Y_t - AX_t - H\xi_{t|t-1}$. The log-likelihood function is iteratively updated using the Kalman filter recursion[7] where closed form solutions for $\hat{\xi}_{t+1|t}$ and $\hat{P}_{t+1|t}$ can be solved for using the Kalman gain.

The system may suffer from a potential endogeneity problem given the theoretical underpinnings of the interest rate policy rule defined by (1) and (2). Endogeneity is addressed using the two step procedure outlined in Chang-Jin et al. (2010). Second lags of inflation and the output gap are used in order to avoid an errors-in-variables problem. The estimation employs a random walk Metropolis-Hastings (MH) Markov chain Monte Carlo (MCMC) algorithm. This is because the conditional likelihood of the candidate draws for this model do not have a closed form solution.

[7] The reader is referred to (Hamilton, 1994, Ch. 13) for a more thorough exposition of the state-space form and the corresponding Kalman filter recursion.

But since the conditional likelihood is proportional to the product of the model likelihood and the prior, it is possible to evaluate the posterior numerically. The conditional posterior density can be expressed as

$$\pi\left(\theta|y_{1:n-1}, y_n\right) \propto \pi\left(\theta|y_{1:n-1}\right) \pi\left(y_n|\theta, y_{1:n-1}\right) \propto \prod_{t=1}^{n-1} \pi\left(y_t|\theta\right) \pi\left(\theta\right) \pi\left(y_n|\theta\right),$$

where $\pi\left(y_n|\theta, y_{1:n-1}\right)$ represents the likelihood and $\pi\left(\theta|y_{1:n-1}\right)$ represents the model prior which are updated with new measurements and candidate densities. Each new candidate draw is based in part on the previous draw. This is because the unobserved state evolves over time based in part on its previous values. The marginal likelihood of a candidate draw can be calculated as the sum of the log likelihood of the model at that draw and the log prior (loosely, the joint probability that the draw came from the prior distribution). This candidate marginal likelihood is compared to the old draw and either accepted or rejected according to a random draw from a uniform distribution. If the new candidate is accepted then it becomes the old draw in the next iteration. If the new candidate is rejected then the old draw is maintained in the next iteration. In this fashion the joint posterior distribution is constructed. The algorithm starts with an initial candidate equal to a maximum likelihood estimate (the posterior mode) of the model. Additionally, the Hessian matrix from the maximum likelihood estimate dictates the variance of the candidate draws chosen.

In order for the initial candidate at the posterior mode to not unduly impact the estimation there is a burn-in period. A total of 500,000 repetitions of the above algorithm are calculated and the first 250,000 are burned, discarded. Preliminary diagnostics of the model indicate that the model converges before the 100,000th iteration. The last 250,000 draws are used to approximate the joint posterior density.

Table 1: Prior distributions

Parameter	Distribution	0.05 pc.	Mean	0.95 pc.
a	Normal $(1.5, 0.25)$	0.5	1.5	2.5
b	Normal $(0.125, 0.13)$	-0.582	0.125	0.832
c	Normal $(0.75, 0.13)$	0.043	0.75	1.457
d	Normal $(0.2, 0.25)$	-0.8	0.2	1.2
$\bar{r}$	Normal $(-0.191, 0.13)$	-0.0418	-0.0191	0.0036
ρ	Normal $(0.5, 0.25)$	-0.5	0.5	1.5
$\sigma^2_{\epsilon_c}$	Inv. Gamma $(2, 1)$	0.355	1.678	4.744
$\sigma^2_{\epsilon_\pi}$	Inv. Gamma $(1, 0.17)$	0.009	0.1188	0.510

3.2 Priors

The prior distributions for the model parameters are chosen either according to previous literature or to reflect specific properties inherit in the data. Table 1 summarizes the prior distributions for all the chosen model parameters. The parameters a and b are set according to Rabanal and Rubio-Ramírez (2005). These are chosen so as to leave the distributions relatively wide. Note the empirical equation is estimated in differences rather than the level of interest rate which is the case for Rabanal and Rubio-Ramírez (2005). Because of this, it is not expected that the posterior distributions will be as wide particularly for the a coefficient on the contemporaneous inflationary gap since the equation is stationary by design. In order to capture a large array of possible values with which to describe past dynamics, priors for c and d are left relatively wide. This is done because different policy data implies differing degrees of persistence and so as to not impose too tight of a parameter space on the meanderings of the algorithm. The constant term, $\bar{r}$, is centered around the unconditional mean of the change in interest rates for all countries. ρ_π is chosen so as to be agnostic regarding the autoregressive parameter. Its distribution is wide enough to even allow for unstable roots in the inflation target. The variance of the policy rule disturbance, $\sigma^2_{\epsilon_c}$, is set according to Scott and Barari (2017). $\sigma^2_{\epsilon_\pi}$ is based from Dossche and Everaert (2005) but is similar to Kozicki and Tinsley (2005) and Smets and Wouters (2005). Collectively these parameter distributions represent a conjugate-normal prior.

3.3 Data

Data used for this estimation is provided from the Federal Reserve Economic Database (FRED) hosted by the St. Louis Federal Reserve bank[8]. The output gap is constructed from the difference in estimated real GDP and an Hodrick-Prescott filter of the same data as a measure of natural output. This series is then converted so that the output gap is a percent. Inflation for all of the countries is calculated from consumer price index estimates[9]. The inflation rates are quarterized. The interest rate data is the policy rate for each country's respective central bank. Each one captures differing degrees of market forces and systematic risks according to the idiosyncrasies of the institution[10]. The sample period for each economy depends on the availability of the data and is not perfectly aligned. The quarterly data is as follows; Australia 1969:3 to 2013:1, Canada 1961:1 to 2014:4, EU 1999:1 to 2013:3, Japan 1960:1 to 2014:4, United Kingdom 1959:1 to 2014:4, and United States 1955:2 to 2015:1.

4 Results

Table 2 summarizes the posterior distributions of the commitment policy empirical model, Equations (25) and (26), for the six economies. Generally speaking the posterior distributions for all the parameters moved away from their priors. This can be interpreted as the priors were wide enough to allow the data to speak for itself. Posterior estimates for a, the coefficient on the inflationary gap, and b, the coefficient on the output gap, indicate that they are important to explaining interest rate dynamics. Except for the estimates of b for Canada and Japan, they are all relatively similar in size as well. There is relatively more variation in c and d for the six economies; this reflects

[8]Data notes indicate that some of the series are originally sourced from OECD database.

[9]It is worth noting that the Federal Reserve is on record as preferring core PCE inflation. Additionally, for the U.S. the Congressional Budget Office provides estimates of natural output. These however are not available for other countries so they are not used in an effort maintain comparability across countries.

[10]All transformed data, MATLAB routine files, and an unpublished appendix with all model details are available from the author upon request.

the differing degrees of persistence inherent in the data. The autoregressive parameter on the unobserved inflation target, ρ, is highly persistent and important for all countries. This indicates that policy makers are reluctant to make relatively large changes to the implicit inflation target and is consistent with Dossche and Everaert (2005), Kozicki and Tinsley (2005), and Scott and Barari (2017).

4.1 Implicit Inflation Target Estimates

One-step ahead forecasts of the state are approximated using the Carter and Kohn (1994) algorithm to sample from the posterior density of the state to capture the degree of probabilistic uncertainty that the model adequately describes the unobserved dynamics[11]. This algorithm draws repeatedly from the posterior density of the state variables over time to construct a probabilistic estimate of the state. Figure 1 contains the 90% credibility interval of the distribution of the inflation target portion of state vector for each economy in alphabetical order. Observed inflation is depicted in black while the estimated inflation target band is in blue. The mean of the credibility interval is denoted by the dotted blue line in the center of the band. The upper bound of the band is the upper 95% and the bottom is the lower 5% of the credibility interval. Generally speaking the bands tend to follow the smoothed time path of inflation for each country. Some bands are broader than others reflecting a greater degree of uncertainty in the data. For example, the Japan (Figure 1d), U.K. (Figure 1e), and U.S. (Figure 1f) estimates have relatively wide bands for the inflation target while Australia (Figure 1a), Canada (Figure 1b), and the E.U. (Figure 1c) have relatively tighter bands. For all countries except Japan, the means of the bands do not reach zero. The lower bound does touch zero for a short while at the end of the series for both U.K. and U.S. This is consistent with previous estimates for the U.S. shown in Scott and Barari (2017). The target at the mean is slightly lower than those reported by Ireland (2007), but there are quite a few notable differences between that study and this estimation. Japan's inflation target distribution is not distinguishable

[11]Not only uncertainty of the model, but also the parameter space.

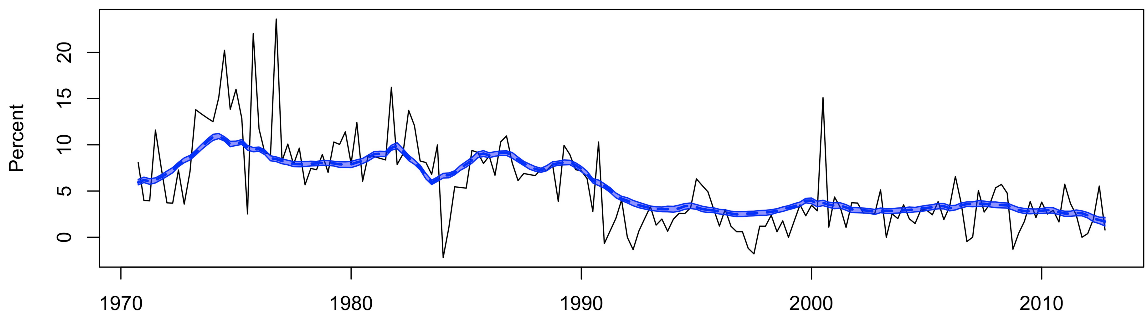
from zero for most of the 2000's, a time period where growth rates in Japan have been close to zero as well. Where observed inflation meets the estimated band can be interpreted as the central bank meeting its inflation target. Persistent periods where observed inflation does not meet the estimated inflation target interval implies that the central bank deviated from its stated goal. This is exemplified in the high inflationary period of the mid to late 1970s for most of the economies in this study. Generally, inflation dynamics are more volatile than the posterior estimates of the implicit inflation target so they tend to oscillate around the estimated range.

4.2 Inflation Rate Policy Deviations

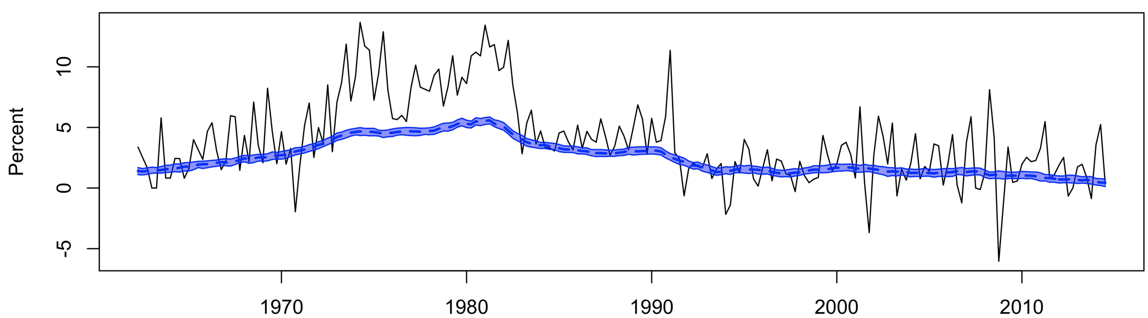
Using the posterior densities, the distribution of the inflation target estimates from above, and realized data, simulations of the policy deviations are calculable. Arguably, it is more intuitive to show the deviations from the inflation simulations alongside observed inflation. Figure 2 is organized similar to Figure 1. Observed inflation is shown in black. The commitment inflation policy credibility interval is represented by the blue bands while the discretion inflation policy credibility interval is represented by the bands in red. As before, the upper bound of each band is the upper 95% and the bottom is the lower 5% of the interval. As with the implicit inflation target, when realized inflation outcomes coincide with the estimated credibility interval then inflation is consistent with the type of policy that generated the band. Since timeless perspective commitment and discretionary policy inherit differing degree of dynamics, the shape of their corresponding credibility intervals is may differ considerably depending upon current and lag values of the output gap and interest rate dynamics. This can cause both commitment and discretionary policy intervals to differ from one another and the observed inflation data for sustained periods.

There are a few conclusions that we can draw from these plots. First, there is an observable significant inflation policy deviation for all countries except the U.K. (Figure 2e) and for prolonged periods prior to 2000. Since the colored band indicates statistically equivalent inflation outcomes

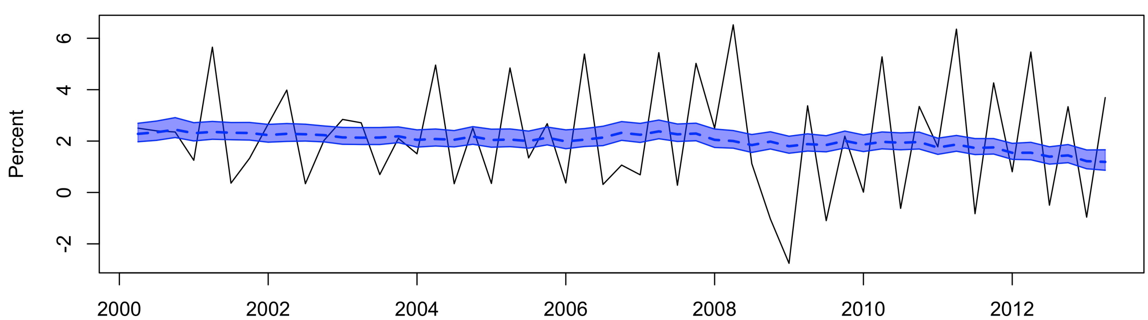
Figure 1: Observed Inflation (black) and Estimated Inflation Target (blue)



(a) Australia

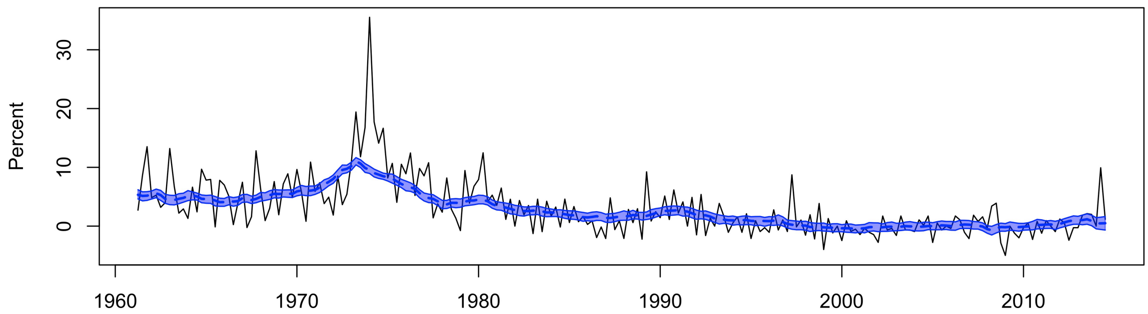


(b) Canada

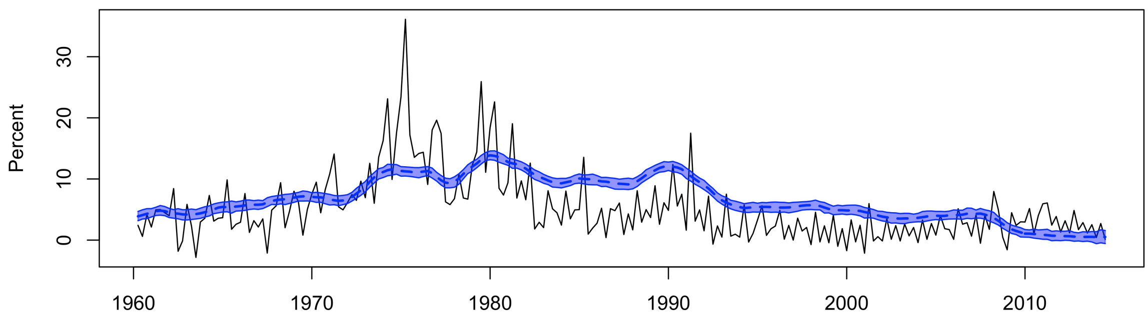


(c) European Union

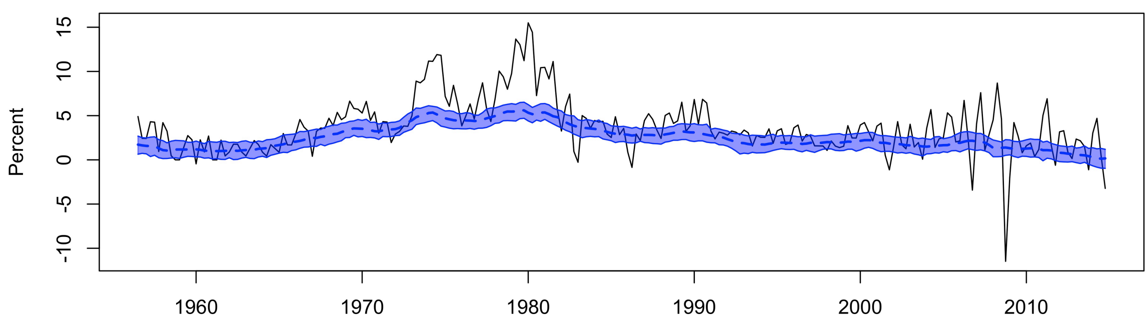
Figure 1: Observed Inflation (black) and Estimated Inflation Target (blue)



(d) Japan



(e) United Kingdom



(f) United States

when the red band is above the blue for a prolonged period then we observe an inflationary policy deviation. When the two colored bands meet the deviation is eliminated. Second for Canada (Figure 2b), E.U. (Figure 2c), U.K. (Figure 2e), and U.S. (Figure 2f) the inflation policy deviation over the entire subsample is relatively small. Australia (Figure 2a) and Japan (Figure 2d) observe a policy deviation for most of their respective sample periods. Third, after 2000 all countries except Australia observe little to no inflation policy deviation. The average inflation deviation over all six economies for the full sample period at the mean of the distributions is 1.3868%. Fourth, when there are sustained and significant inflation policy deviations, inflation policy tends to follow more closely with discretionary based policy regimes. The exception to this is the brief period around 1990 for the U.S. (Figure 2f).

The first three columns of Table 3 highlight the shrinking of the average inflation policy deviation that we see in the posterior estimates. This table is divided into four subsections; full sample period, sample prior to 1980, sample prior to 2000, and the sample post 2000[12]. The first three columns express the average inflation policy deviation as a distribution that can loosely be thought of as a credibility interval around the mean inflation deviation. All four sample periods highlight the degree of variation within the deviation estimates across economies. Australia and Canada have considerably tighter distributions, while E.U., Japan, U.K., and U.S. have relatively wider distributions. Prior to 2000 the average inflation policy deviation at all points in the distribution (5%, mean, and 95%) is higher than the full sample average. Likewise, leading up to the high inflation periods worldwide, prior to 1980 we see average inflation policy deviations at all points lower than the full sample average. Most of the growth in average inflation policy deviation occurs during the declining inflation years of the later 1980s and 1990s. Post 2000 the average inflation policy deviation is pulling the overall average downward, and this is seen in the bottom portion of the table. The average inflation policy deviation at all points in the distribution is below the full sample average. This implies that the discretion/commitment policy tradeoff from an inflationary

[12]The E.U. is not included in the subsample calculations since the monetary union started in 1999.

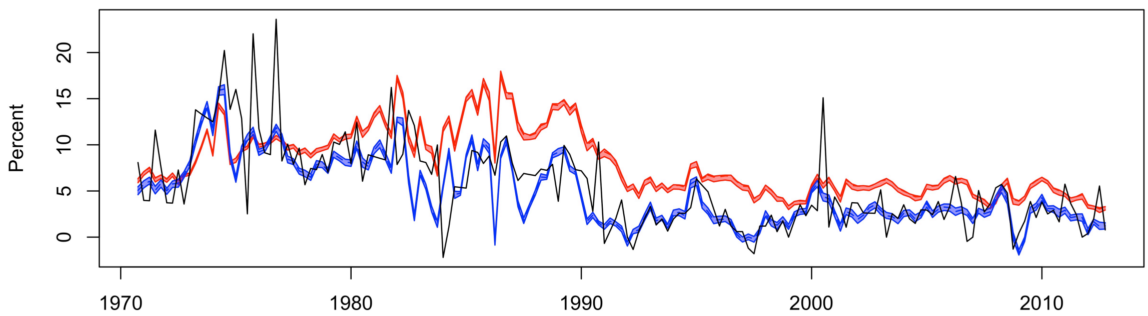
perspective is less of a concern before and during the global financial crisis. These results and the evidence shown in Figure 2 highlights that the inflation policy deviation is effectively zero for all countries except for Australia so far in the twenty first century.

4.3 Interest Rate Policy Deviations

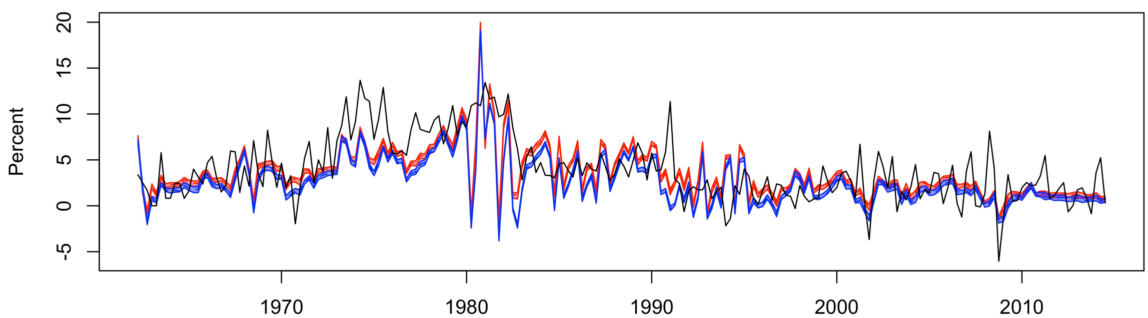
In the same fashion that discretionary inflation is on average higher than commitment inflation, discretionary interest rate policy is on average lower than commitment interest rate policy. Because of the connection between inflation and the interest rate through the Fisher equation, the dynamic inconsistency of monetary policy also induces an interest rate policy deviation that mirrors the inflation policy deviation. This interest rate deviation is observed in Figure 3. As with Figure 1 and Figure 2, each subfigure shows observed data and posterior simulations for each economy in alphabetical order. The observed policy rate is in black, the commitment policy band is in blue, and the discretionary policy band is in red. Figure 3 is also interpreted in the same way as Figure 2. When the policy rate coincides with one or more of the estimated credibility intervals then interest rate policy is consistent with the type of policy regime. Furthermore, when the two estimated policy regimes overlap and diverge is of importance as this implies an opportunity cost of policy.

There are a few conclusions that can be drawn from these plots. First, on average the interest rate policy that would occur under a strategy of commitment is statistically higher than its discretionary equivalent for all countries except the U.K. (Figure 3e). This implies that discretionary policy is more often loose than tight. Policy makers that act in a counter-cyclical discretionary fashion are more often dovish than hawkish in terms of interest rate policy. This is consistent with the inflation results from Figure 2. Additionally, it is also consistent with the discretionary asymmetric preferences literature mentioned above. Second, the interest rate policy deviation is either reduced or eliminated for Canada (Figure 3b) and the U.S. (Figure 3f) post 2000. For Japan (Figure 3d), the interest rate policy deviation is eliminated around 1995. For Canada (Figure 3b) and the E.U.

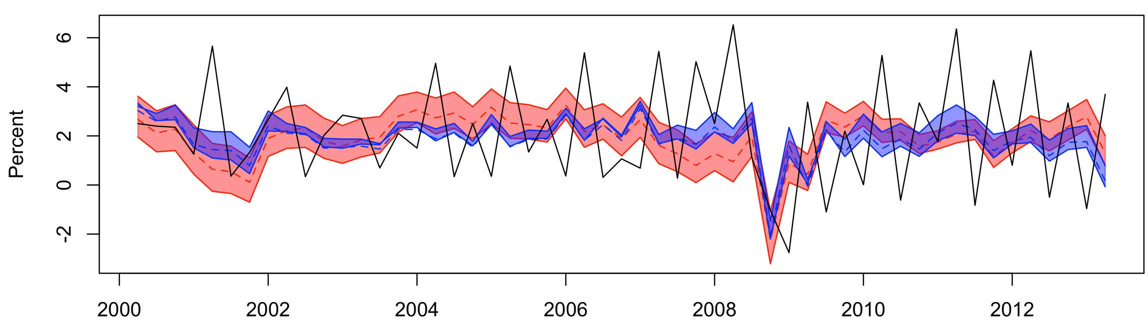
Figure 2: Observed Inflation (black) Along with Discretionary (red) and Commitment (blue) Policy



(a) Australia

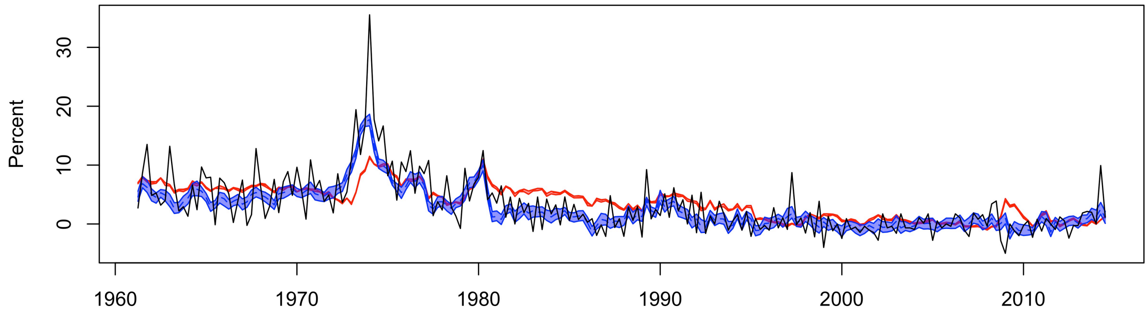


(b) Canada

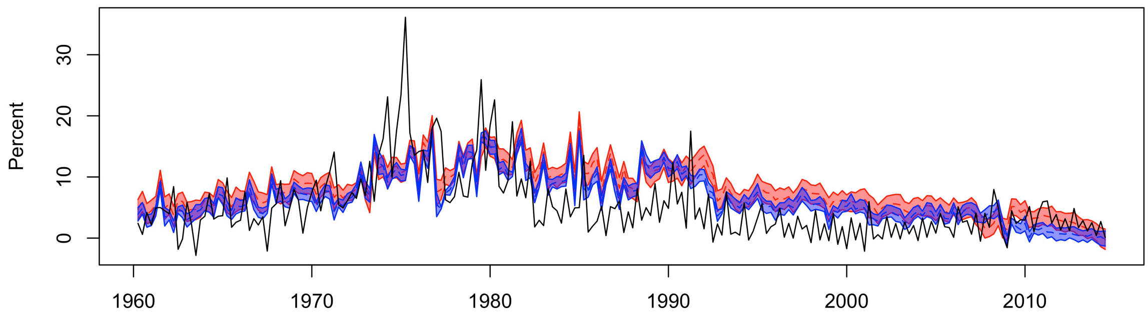


(c) European Union

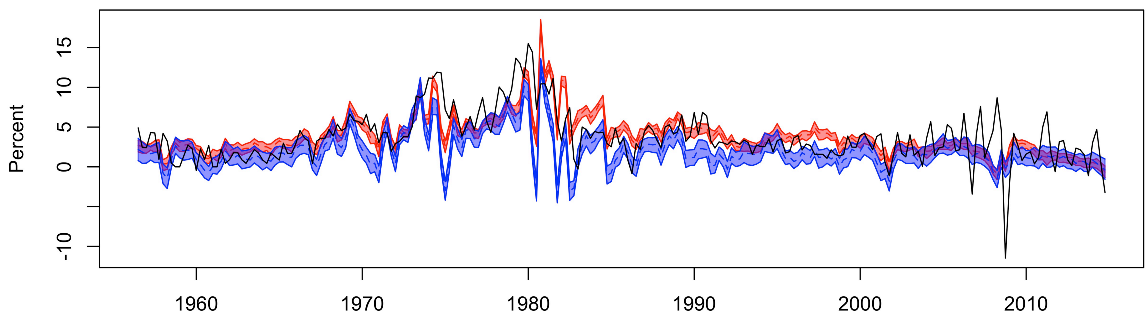
Figure 2: Observed Inflation (black) Along with Discretionary (red) and Commitment (blue) Policy



(d) Japan



(e) United Kingdom



(f) United States

(Figure 3c) the policy deviation is not eliminated until the global financial crisis, around 2007 for Canada and about 2009 for the E.U. It is reduced at the end of the 1990's for Australia (Figure 3a), but it still remains post 2000. These results largely mirror the inflation results. Third, the average interest rate policy deviation for all six economies over the full sample period at the mean of the distributions is 1.9927%. This reflects the Taylor principle holding for these countries.

One important feature of interest rate policy for developed economies in the past ten years is when interest rates are low and near an effective zero lower bound (ZLB). When interest rates are sufficiently low and the monetary authority has committed an intolerance for negative nominal interest rates, the economy can experience the effects of a liquidity trap. Not all of the central banks examined in this study committed to such a policy stance. Nevertheless, the relatively sharp reduction of policy rates and hesitance to move them in the wake of the global financial crisis of 2007-2009 is still important. While this exercise does not explicitly account for the ZLB policy stances, it is designed to minimize its effect on the approximation of the joint posterior density. Since the interest rate rule is estimated in first differences and not levels. The time period corresponding to ZLB policy at most represents a reduction in the volatility of a stationary series. For most of these economies this not likely to bias the empirical results because the time period is so small relative to the overall sample size. The exception to this is Japan which hit a ZLB near 1994 close to the start of its low growth phase.

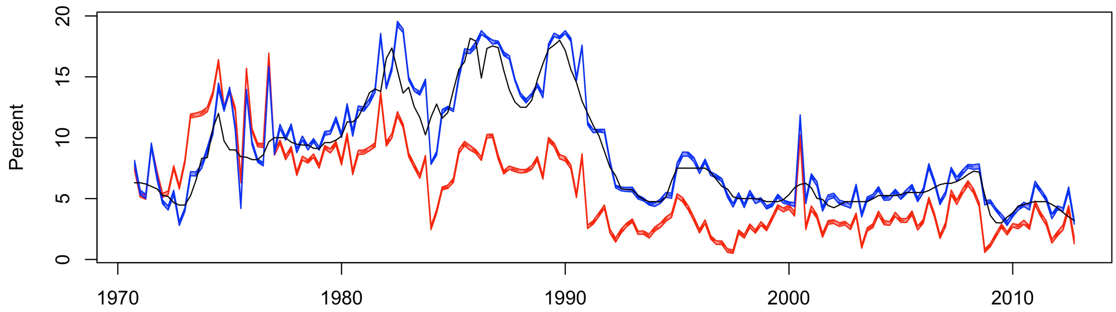
The last three columns in Table 3 show the dynamics of the interest rate policy deviation for the six economies over the full sample period, before 1980, before 2000, and after 2000. Similar to their inflation counterparts, these columns can be interpreted as credibility intervals around the mean deviation estimate. The relatively tight inflation deviation interval for Australia and Canada is reflected in their respective interest rate deviation interval. Alternately, the wider interval for E.U., Japan, U.K., and U.S. are also born out in the interest rate bias interval. Exiting the high inflation policy years for most of these countries is also shown in the interest rate side of the table. The average deviation dips for all economies prior to 1980 relative to the full sample at all points in the distribution (5%, mean, and 95%). Prior to 2000, the average interest rate policy deviation at all points in the distribution is higher than the full sample average. Post 2000, the average decreases pulling the average down for all countries. For all countries except Australia interest rate policy is not substantially different than what commitment policy would prescribe during and coming out of the global financial crisis. Whether this is due to low inflation expectations or the lasting effects of ZLB policy or some other cause is not identifiable in this framework.

4.4 Inflation Deviations and Targeting Behavior

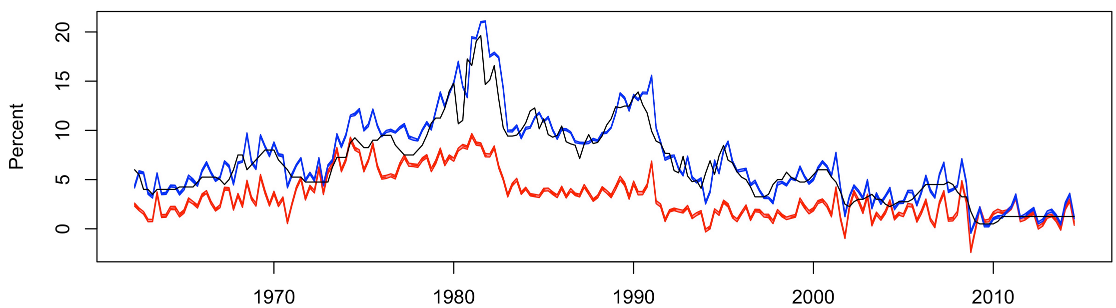
As mentioned above, the reduced form parameters a and b govern the degree of inflation versus output gap targeting behavior from the monetary authority. While both aversion parameters are positively related to the policy rule (Equation 19), the relationship with the average inflation bias (Equation 24) is somewhat more nuanced. An increase in inflation targeting behavior (a) puts upward and downward pressure on differing terms in the average inflation bias calculation while an increase in output gap stabilization (b) increases the counter-cyclical relationship between the output gap and average inflation deviations. To show this effect more clearly, Figure 4 contains three-dimensional plots that display simulated values of the average inflation deviation calculation over a grid of possible inflation and output gap targeting parameter values. The domain of the grid space is designed to show the effect of both reasonable and extreme possible values of these parameters. The remaining parameters in the calculation are represented by the posterior means of the parameter distributions and empirical data is used in the calculation for each economy.

The shape of the simulation grids generally shows an upward trend in the average inflation bias calculation as the degree of inflation targeting decreases. Additionally, as the degree of output gap targeting increases so too does the average inflation deviation for all values of a. This effect is more pronounced for relatively smaller values a. Under a regime of relatively strict inflation targeting, a is relatively high, and the degree of output gap sensitivity is of little importance to the

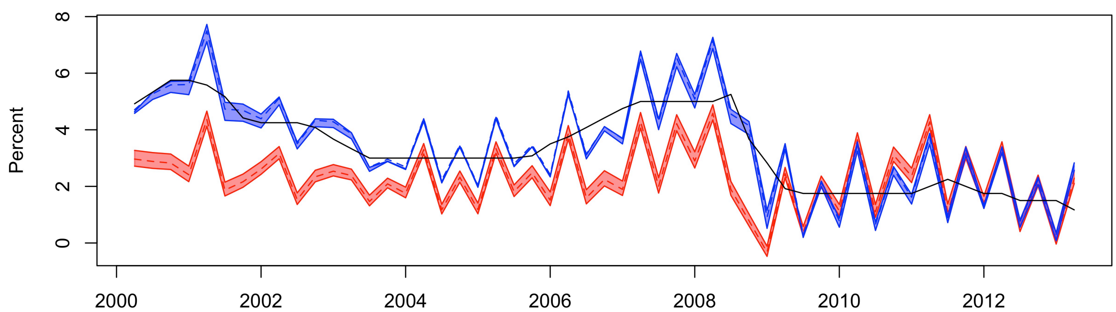
Figure 3: Observed Interest Rate (black) Along with Discretionary (red) and Commitment (blue) Policy



(a) Australia

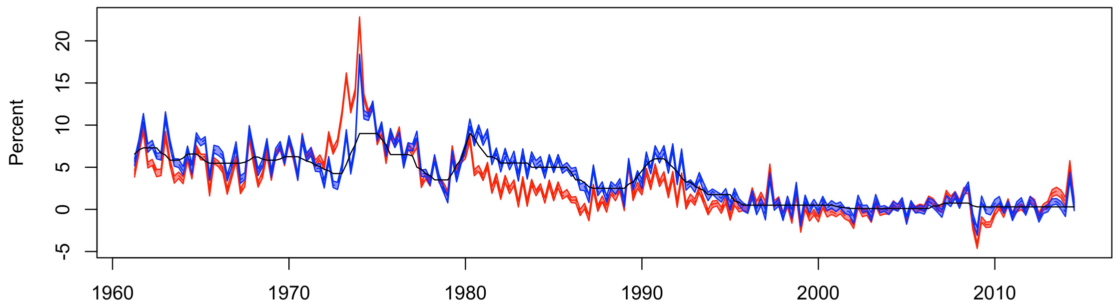


(b) Canada

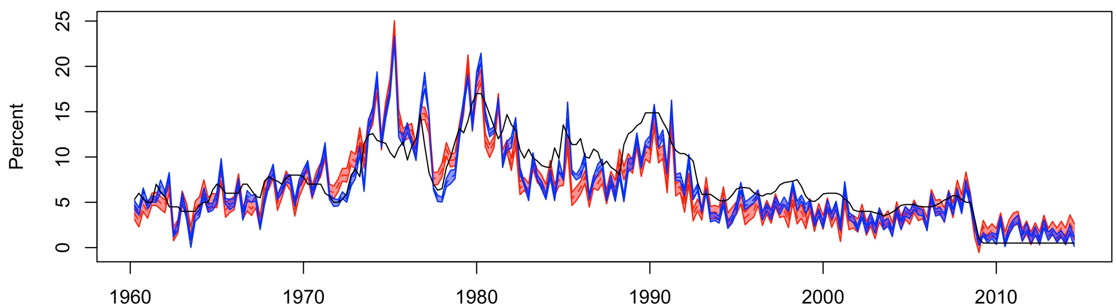


(c) European Union

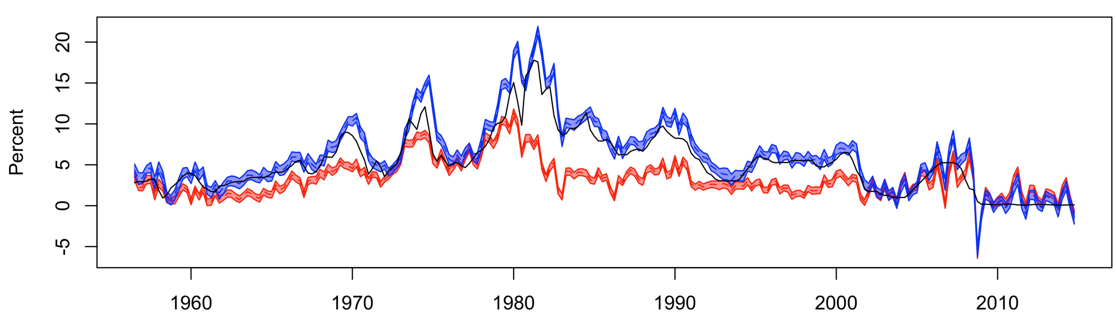
Figure 3: Observed Interest Rate (black) Along with Discretionary (red) and Commitment (blue) Policy



(d) Japan



(e) United Kingdom



(f) United States

Table 2: Posterior distributions - Timeless perspective policy

	Australia - 1969:3 to 2013:1			Canada - 1961:1 to 2014:4			European Union - 1999:1 to 2013:3		
Parameter	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
a	0.5042	0.5048	0.5053	0.4777	0.4796	0.4794	0.4483	0.4487	0.4491
b	0.6576	0.6715	0.6839	0.0989	0.1364	0.1336	0.3513	0.4006	0.4484
c	0.1352	0.1397	0.1438	-0.0342	-0.0230	-0.0239	-0.0795	0.0644	0.204
d	0.5452	0.5673	0.5871	0.2132	0.2168	0.2168	-0.3986	-0.246	-0.0978
$\bar{r}$	0.2891	0.3136	0.3354	0.0938	0.0943	0.0943	0.171	0.2044	0.2388
ρ	0.9107	0.9121	0.9133	0.7528	0.7650	0.7641	0.898	0.8985	0.899
$\sigma^2_{\epsilon_c}$	0.7137	0.8331	0.9668	6.0400	6.5390	6.5413	0.6808	0.8292	0.9821
$\sigma^2_{\epsilon_\pi}$	0.1532	0.1600	0.1662	0.0212	0.0278	0.0273	0.0966	0.0975	0.0983
Marg. Likelihood	-619.91	-618.48	-617.98	-699.1589	-697.1665	-696.9224	-138.2569	-136.6605	-136.4521

	Japan - 1960:1 to 2014:4			United Kingdom - 1959:1 to 2014:4			United States - 1955:2 to 2015:1		
Parameter	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
a	0.4981	0.4985	0.4989	0.4992	0.4995	0.4997	0.5460	0.5537	0.5606
b	0.3156	0.3198	0.3248	0.5615	0.5867	0.6121	0.5230	0.5260	0.5295
c	0.1024	0.1034	0.1042	0.0969	0.0980	0.0991	0.0850	0.0867	0.0886
d	0.8647	0.8954	0.9217	-0.0531	0.0777	0.2078	0.3822	0.3933	0.4032
$\bar{r}$	0.1015	0.1021	0.1027	0.1849	0.5241	0.8651	0.1109	0.1127	0.1143
ρ	0.9052	0.9073	0.9091	0.9003	0.9021	0.9039	0.9088	0.9103	0.9116
$\sigma^2_{\epsilon_c}$	0.5020	0.5875	0.6872	2.0012	2.0035	2.0059	0.8095	0.9444	1.0957
$\sigma^2_{\epsilon_\pi}$	0.1015	0.1022	0.1027	0.1001	0.1016	0.1030	0.1247	0.1289	0.1326
Marg. Likelihood	-619.97	-618.52	-618.01	-1005.2474	-1003.8067	-1003.2954	-708.5359	-707.0709	-706.5573

Joint posterior distributions are constructed from the random-walk Metropolis Hastings MCMC algorithm. The policy rule equation is given by (19), $\Delta i_t = c\bar{r} + a\{\pi_t - \pi^*_t\} + \Delta\pi^*_t + b\Delta x_t + c\{i_{t-1} - \pi^*_{t-1}\} - d\{\Delta i_{t-1} - \Delta\pi^*_{t-1}\} + \epsilon^c_t$. The remaining parameters correspond to (4). These equations are estimated jointly.

Table 3: Posterior Estimates of Timeless Perspective and Discretionary Policy Deviations

Full Sample

	Average inflation deviation			Average interest rate deviation		
	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
Australia	2.9980	3.2085	3.3003	2.9959	2.9917	2.9947
Canada	0.8088	0.8643	0.9241	3.8193	3.8034	3.7306
European Union	-0.6896	0.0929	0.6726	1.3043	1.2366	0.9497
Japan	0.3470	1.2119	1.8519	0.6846	0.7482	0.8174
United Kingdom	0.5150	1.1880	1.6649	-0.1900	0.1180	0.4135
United States	1.2459	1.7551	2.1778	2.6781	2.8227	2.9806

Prior to 1980

	Average inflation deviation			Average interest rate deviation		
	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
Australia	0.1916	0.5000	0.5308	-0.7223	-0.7175	-0.6952
Canada	0.6968	0.7053	0.7723	3.1137	3.1758	3.2005
European Union	.	.	.	.	.	.
Japan	-0.3426	0.4376	1.1278	-0.0494	0.0076	0.0920
United Kingdom	0.4352	1.0514	1.4715	-0.5735	-0.2815	0.0229
United States	0.8867	1.4121	1.8473	2.1120	2.2692	2.4267

Prior to 2000

	Average inflation deviation			Average interest rate deviation		
	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
Australia	3.3673	3.5112	3.6134	3.4294	3.4305	3.4321
Canada	0.9626	0.9966	1.0746	4.6424	4.7253	4.7412
European Union	.	.	.	.	.	.
Japan	0.6065	1.3896	2.0530	0.8963	0.9617	1.0371
United Kingdom	0.5385	1.2158	1.7009	-0.0868	0.2063	0.5114
United States	1.6374	2.1412	2.5536	3.4381	3.5848	3.7364

Post 2000

	Average inflation deviation			Average interest rate deviation		
	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
Australia	2.2586	2.4870	2.5446	2.0067	2.0140	2.0143
Canada	0.4503	0.5041	0.5254	1.3969	1.4440	1.4600
European Union	.	.	.	.	.	.
Japan	-0.1506	0.5996	1.2854	0.1284	0.1874	0.2403
United Kingdom	0.4700	1.1141	1.5482	-0.4681	-0.1199	0.1498
United States	0.1106	0.6356	1.0879	0.4741	0.6128	0.7888

Table 4: Welfare Loss Posterior Estimates - Percent Loss Relative to Pre-Commitment

Full Sample

	Percent Welfare Loss		
	0.05 pc.	Mean	0.95 pc.
Australia	0.3499	0.5316	0.5436
Canada	0.1734	0.6640	0.7158
European Union	5.2160	6.5514	8.0343
Japan	4.1788	4.2394	4.4419
United Kingdom	2.6610	3.1294	4.0001
United States	5.6375	6.1863	6.2373

Prior to 1980

	Percent Welfare Loss		
	0.05 pc.	Mean	0.95 pc.
Australia	0.0810	0.1356	0.3303
Canada	0.3811	0.4805	0.6328
European Union	·	·	·
Japan	5.6263	5.8807	6.0664
United Kingdom	2.9492	3.2407	3.3295
United States	15.1848	15.4121	15.5886

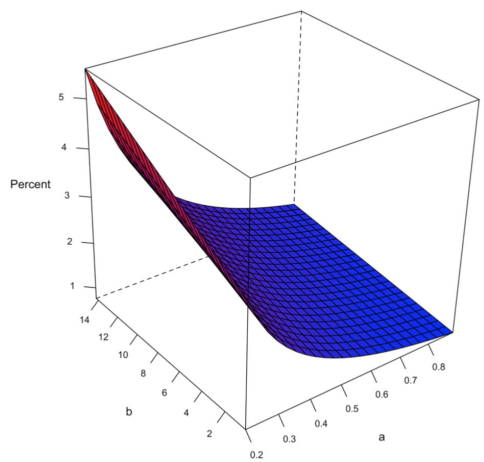
Prior to 2000

	Percent Welfare Loss		
	0.05 pc.	Mean	0.95 pc.
Australia	2.3032	2.3201	2.4320
Canada	0.7319	1.2562	1.2747
European Union	·	·	·
Japan	3.3231	3.5893	3.6454
United Kingdom	2.3556	2.9983	3.9769
United States	10.7369	10.7423	11.3331

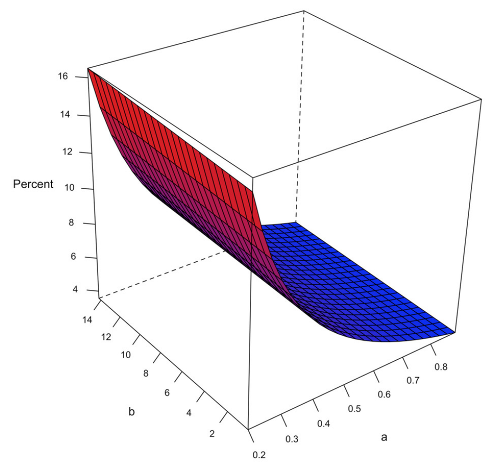
Post 2000

	Percent Welfare Loss		
	0.05 pc.	Mean	0.95 pc.
Australia	7.2317	7.7813	8.1546
Canada	1.5132	1.8303	2.0888
European Union	·	·	·
Japan	9.7504	11.0351	11.2126
United Kingdom	4.1141	4.2830	5.4446
United States	0.4346	1.0793	1.7846

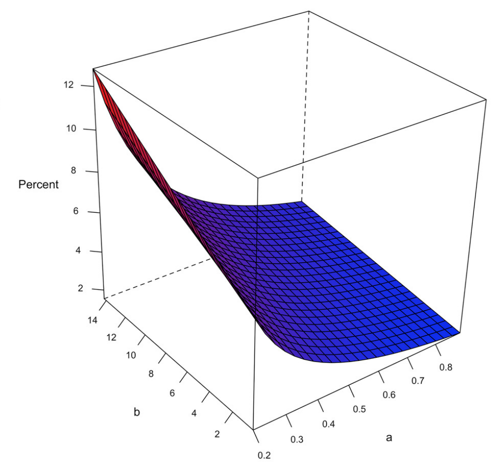
Figure 4: Percent Deviations Relative to Pre-commitment for Varying Degrees of Inflation and Output Gap Targeting



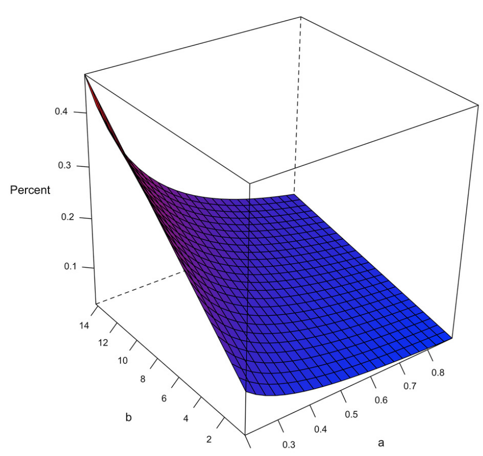
(a) Australia



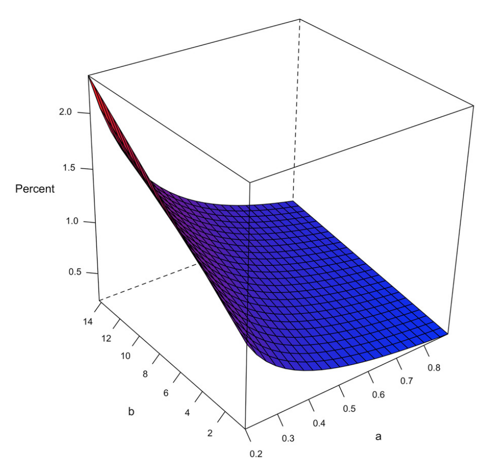
(b) Canada



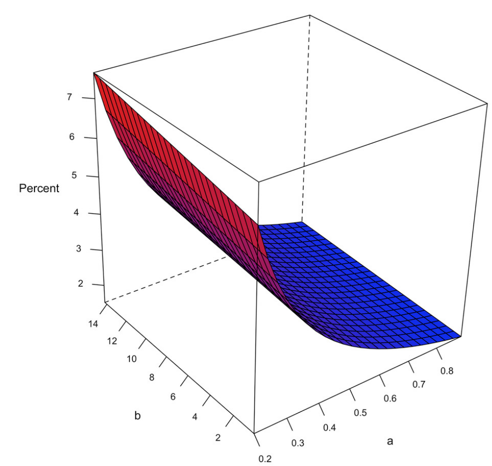
(c) European Union



(d) Japan

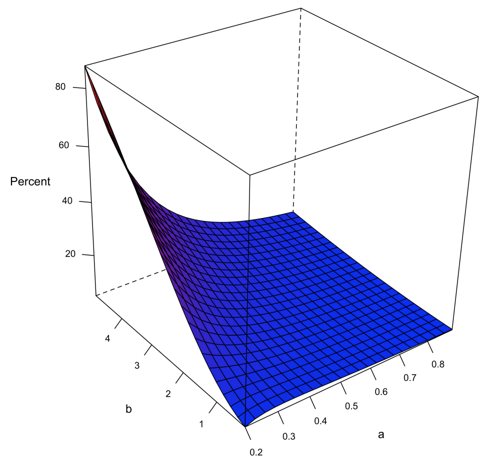


(e) United Kingdom

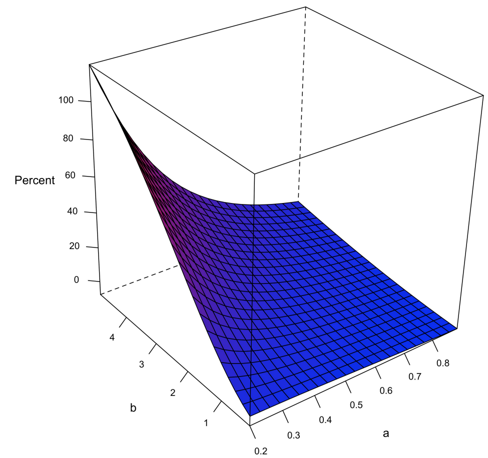


(f) United States

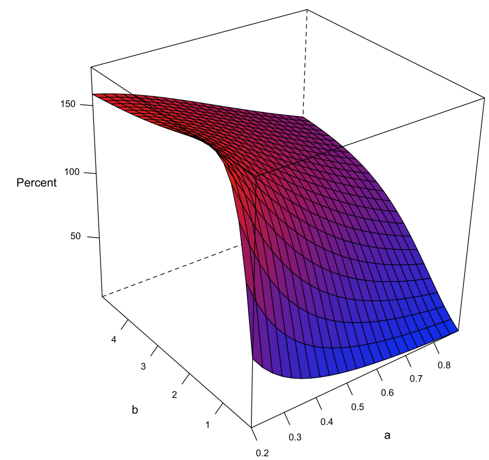
Figure 5: Percent Unconditional Loss Relative to Pre-commitment for Varying Degrees of Inflation and Output Gap Targeting



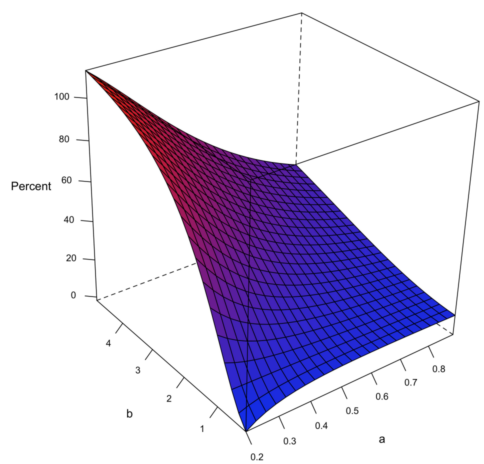
(a) Australia



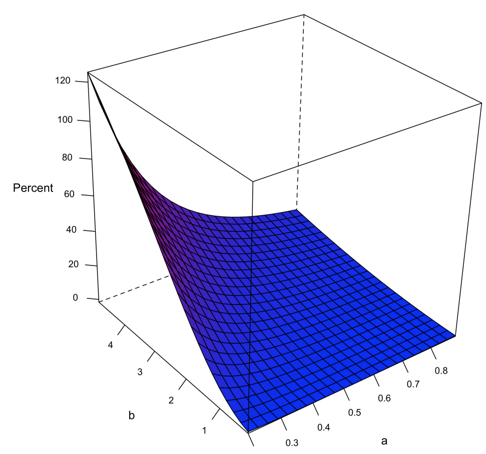
(b) Canada



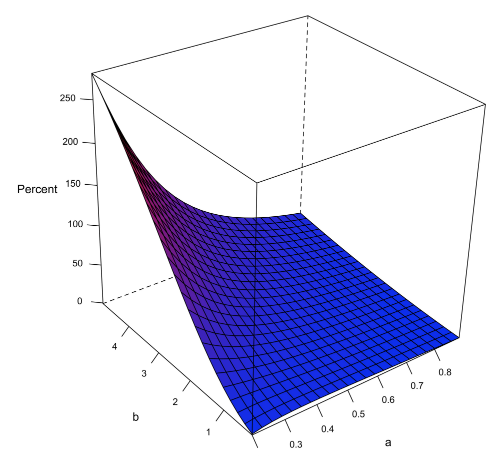
(c) European Union



(d) Japan



(e) United Kingdom



(f) United States

average inflation bias calculation. The minimum for all countries is shown to be the combination of high inflation and low output gap sensitivity. The highest simulated average inflation deviation occurs with low inflation priority and a high degree of output gap stabilization. So long as the central bank does not take its eyes off of the inflationary prize, the penalty for discretionary policy from a potential inflation bias perspective is relatively low regardless of preferences for output gap stabilization.

4.5 Relative Welfare Loss and Targeting Behavior

Another method of measuring the effect of commitment versus discretionary monetary policy is the calculation of welfare loss under the two competing regimes as defined above. While much has been written on this subject, Walsh (2003) implements a relative welfare loss metric where loss under discretion is measured as a percent of loss under commitment. Table 4 and Figure 5 summarize this metric in percent terms for the six different economies but along different dimensions. Table 4 measures the posterior density of the percent of welfare loss relative to timeless perspective commitment policy over the full sample and same subsamples as Table 3. Figure 5 replicates the simulation exercise of Figure 4 but instead of percent of policy deviations we have relative welfare loss along the vertical axis.

The results in Table 4 are similar in magnitude to the loss calculations reported in Walsh (2003). The commitment policy formulation used in this model inherits the same speed-limit dynamics, though there are some notable differences. The subsamples tell a more dynamic picture of relative welfare loss than the full sample estimates. The differences in interest rate policy is the primary determinant of these welfare loss calculations. This is true not only of the difference between the credibility intervals but also the variance of these intervals. The high inflation period of the late 1970s and early 1980s increases the relative loss calculation indirectly via the policies leading up to or following the inflationary spike. This is seen the pre-1980 and pre-2000 subsample estimates.

The post 2000 subsample produces relatively higher percents mostly due to the fluctuations in the competing interest rate policies. Australia, Canada, Japan, and the U.K. all have higher relative welfare losses than the full sample estimates during this time. Australia's post 2000 interest rate policy deviation is credibly different. This is consistent with the findings above. Canada has credibly different interest rate policy for the first half of this subsample period. Japan and the U.K. do not have credibly different interest rate policy for this subsample, but the upper bounds and lower bounds of their discretionary policy credibility intervals exceed the limits of the upper bounds and lower bounds of their respective commitment credibility intervals. This is not the case for the U.S. post 2000s, which has a posterior distribution lower than the full sample estimates. The upper bounds and lower bounds of discretionary interest rate policy is fully contained by the upper bounds and lower bounds of the commitment interest rate credibility interval.

Figure 5 contains simulations for full sample relative welfare loss at the mean of the joint posterior distribution for different possible values of the inflation and output gap targeting parameters (a and b respectively). The parameters a and b are the same parameters as found in Figure 4. In the case of the output gap targeting parameter, the domain does not extended as far as the in Figure 4 because of the exploding nature of welfare loss ratio. In all cases the hyperplane appears to hit a maximum when the central bank strongly weights output gap stabilization and places little weight on inflationary movements. In all cases the minimum appears with relatively low weight placed on output gap variations. The shapes of these plots are consistent with the theoretical literature on rules versus discretion which suggests a societal loss when the monetary authority deviates from a pre-committed rule to focus on output/unemployment targets. Similar to Figure 4, when the central bank focuses on strict inflation targeting the percent loss above commitment policy tends to be the lowest (with the exception of Japan). The E.U. (Figure 5c) and Japan (Figure 5d) are two special cases. The curvature in shape for the E.U. case may be a function of the limited amount of data available. Additionally, the Japan simulation implies some slight concavity that is not present in the other simulations. This may be the result of the slow growth and extremely stable interest rate policy over the past 25 years. In all cases when the monetary policy maker has little regard for inflation stabilization the tradeoff between relative welfare loss and output gap targeting is the greatest.

5 Conclusion

This paper employs a familiar New-Keynesian framework to solve the monetary planner's problem under both commitment (timeless perspective) and discretion. Additionally, the planner's unobserved implicit policy targets are assumed to vary over time. Optimal discretionary and commitment policy interest rate and inflation rate rules are solved. The model is estimated using Bayesian methods in order to approximate a joint posterior density for the model parameters. The intrinsic inflation target is also sampled from the posterior distribution of the state. The posteriors are then used to simulate for interest rate and inflation rate policies under the two regimes. The theoretical inflation policy deviation is shown to be a function of lagged values of the output gap, interest rate, and the implicit inflation target.

Results indicate that the inflation policy deviation was historically more important to policy makers than it is today. This deviation has diminished, and in the case of Canada, the E.U., Japan, the U.K., and the U.S., is not significantly different from zero after 2000. The time inconsistency of optimal policy also induces an interest rate policy deviation. Commitment policy is shown to be on average higher than discretionary policy. This implies that monetary policy makers tend to be more dovish when they act in a discretionary fashion. The interest rate policy deviation is either reduced or eliminated during and exiting the global financial crisis. Simulations of both the average policy deviation as well as the welfare loss of discretion relative to commitment generally are greatest when the central bank places low relative weights on inflation targeting behavior and exhibit a higher preferences for output stability. These results are in keeping with the larger literature on rules versus discretionary policy making.

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