How much better is commitment policy than discretionary policy? Evidence from five developed economies

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Abstract

Much has been written on how an active central bank produces inflation outcomes above and beyond what commitment policy would produce. This paper contributes to this body of literature by simulating from the state estimates of both commitment and discretionary policy equilibria in a familiar dynamic New-Keynesian framework. Optimal interest rate and inflation rate policies are derived under the two regimes for five developed economies. The model is estimated using Bayesian methods employing a random-walk Metropolis-Hastings algorithm. The draws from the joint-posterior distribution are used to simulate for the optimal inflation and interest rate policies for each of the countries. Results suggest that the simulated inflation induced by discretionary policy is not significantly different from commitment policy after 2000 for four of the five countries (including the U.S.). Simulated commitment interest rate policy is on average 2.1% higher at the center of the distribution, suggesting that discretionary interest rate policy is on average more often loose compared to commitment interest rate policy.

JEL Classification: E31, E52, E58, E61

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1 Introduction

The inconsistency of optimal monetary policy is a phenomenon that has been well studied in the theoretical monetary policy literature. Kydland and Prescott (1977) discuss how the central bank, when it has discretionary power, can seek to stabilize prices, and the outcome will result in an equilibrium that is optimal but inconsistent. Put another way, the central bank in an effort to target an unemployment rate above the natural rate induces a level inflation above the level society would prefer. Barro and Gordon (1983) expand further on this to suggest that the monetary authority seeking to surprise the public induces an inflation bias. However, in this model society recognizes that the monetary authority has an incentive to deceive them, so the inflation bias is an average level of inflation above how private sector agents formulate expectations. This definition implies that either discretionary or commitment policy rules could in theory produce an inflation bias. The notion discussed by Kydland and Prescott (1977) is somewhat different describing the difference between discretionary and commitment policy outcomes in a control theory framework[1].

Much of the recent empirical literature analyzing inflation outcomes models monetary policy preferences as asymmetric in functional form. This so-called asymmetric preferences literature relaxes the a priori assumption that the policy maker responds with an equal sense of urgency during expansionary and recessionary phases. This notion first appears in the literature by Cukierman and Gerlach (2003), Nobay and Peel (2003), and Ruge-Murcia (2003*a*), where it is shown that policy makers targeting natural output or natural unemployment can induce an inflationary bias by their own response asymmetry. Cukierman and Gerlach (2003) shows that even if the central bank targets the nominal level of employment, a bias is produced due to the uncertainty around economic conditions. It should however be noted that this type of bias is not strictly due to the time inconsistency of optimal decisions, but also because the policy maker's preferences are warped.

Surico (2007) employing the aforementioned asymmetric preferences framework estimated an

[1] For the remainder of this paper the term 'inflation bias' is used more to describe the difference in policy rule outcomes and not in the classic Barro and Gordon (1983) sense.

average inflation bias of approximately 1.5% for the U.S over the period corresponding to the great moderation[2]. Anderson, Kim and Yun (2010) employ a projection method approach in order to examine discretionary and commitment policy. They show that for plausible values of the model parameters the difference between discretionary and commitment policy should be somewhere between 1% to 6%. Billi (2011) in a calibrated model shows that when the central bank commits at least to an inertial Taylor rule the inflation bias is eliminated[3]. These empirical analyses however are effectively measuring an average inflation bias. This method may not be appropriate if the inflation bias is non-stationary over time. Additionally, it tells us little about how the bias might change when the fundamentals of the economy vary.

The unprecedented discretionary monetary policy actions taken by central banks around the world in response to the great recession implies that a deviation from commitment policy should be observed empirically. Since optimal commitment policy does not necessarily involve zero inflation and zero output gap (Kirsanova et al.,2009), the inflation bias is not just discretionary policy inflation rules above and beyond zero inflation. Additionally, commitment policy inflation is not necessarily zero inflation. If the central bank's policy targets are not assumed to be constant over time, commitment policy inflation outcomes inherit dynamics that are statistically different from zero more often than not. When does discretionary policy yield different inflation outcomes for society than what commitment policy would prescribe?

This paper contributes to the literature on optimal monetary policy in three unique ways. First, commitment (timeless perspective) and discretionary policy interest rate and inflation rate rules are solved for in a dynamic equilibrium model with a time-varying inflation and interest rate target. Since these targets are not constant over time,[4] it is shown that relaxing this assumption produces additional dynamics in the optimal policy rules and not just imposed ad hoc. These dynamics

[2]Approximately mid 1980's to mid 2000's.

[3]Its important to note however that both Anderson, Kim and Yun (2010) and Billi (2011) are strictly numerical and not employing observed data.

[4]See Dossche and Everaert (2005), Ireland (2007), Leigh (2008), and Scott (2016) for a more thorough analysis of this hypothesis.

are more pronounced under a strategy of commitment rather than discretion. The commitment interest rate policy rule nests the discretionary interest rate policy rule as special case. Second, this model is confronted with data for five developed economies; Australia, Canada, Japan, United Kingdom, and United States. It is estimated using Bayesian methods employing a random-walk Metropolis-Hastings MCMC algorithm. Joint posterior distribution estimates are provided for all countries. Finally, the posterior distribution of the parameters as well as estimates from the state distribution are used to simulate and produce estimates of both commitment and discretionary policy over time. Since the distribution of the parameters is used, then the simulations produce a distribution of possible inflation and interest rate paths that can be interpreted similar to confidence bands from impulse response functions. This provides a more accurate picture of how policy could hypothetically evolve under the different regimes.

Estimations suggest some important results. First, there are multiple periods of observable significant significant deviations between the two inflation policies prior to 2000 for all countries except the United Kingdom. Post 2000, this deviation disappears for all countries except Australia. The deviation decreases after 2000 at the mean by as little as 0.5% (Canada) and as much as 1.5% (U.S.). The average inflation policy deviation for all five countries at the mean is about 1.64% over the full sample period. Second, interest rate policy simulations imply an interest rate policy deviation as well. Discretionary interest rate policy is on average lower than commitment policy implying that discretionary policy is more often loose than tight (evidence of asymmetric preferences). This interest rate bias largely mirrors the inflation policy deviations due to the underlying Fisher effect. The interest rate bias is present for all countries except the U.K. prior to 2000. Australia's interest rate deviation is still present after 2000. Japan's interest rate deviation disappears during the mid 1990's (the beginning of its low growth phase). Canada's interest rate deviation disappears during the period corresponding to the global financial crisis.

The remainder of this paper is organized as follows. Section 2 details the model and solution under both discretion and commitment (timeless perspective policy). Section 3 explains the estimation procedure and discusses the data used. Section 4 summarizes the findings of the empirical estimation and simulation. Section 5 concludes with a brief discussion.

2 Model

The economy is assumed to evolve according to the canonical form of a standard log-linearized dynamic New-Keynesian model. The consumer's forward-looking Euler equation is given by (1). Here x_t is the deviation in output under sticky prices, (y_t), from output under flexible prices, (y_t^n) or $x_t \equiv y_t - y_t^n$. The expectations operator, E_t, denotes conditional expectations formed using all available information at time t. σ represents the inverse of the intertemporal elasticity of substitution which summarizes the rate of substitution between consumption today versus consumption tomorrow. The observed interest rate is given by i_t and the observed inflation rate is defined by π_t. ϵ_t^d is an error term that is assumed orthogonal to any rational expectations errors and follows $\epsilon_t^d \sim (0, \sigma_d^2)$. As a whole Equation (1) summarizes the demand side of the economy.

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} \right) + \epsilon_t^d \tag{1}$$

The supply side of the economy is governed by a forward-looking New-Keynesian Phillips curve (NKPC), Equation (2). As in Equation (1) π_t, x_t, and E_t are the inflation rate, output gap, and conditional expectations operator respectively. β is the discount factor of the consumer's utility maximization problem. κ is a reduced form parameter that is a function of the Calvo (1983) price adjustment lottery. Finally, ϵ_t^s represents a cost-push inflationary shock and once again is orthogonal to any rational expectations errors and evolves according to $\epsilon_t^s \sim (0, \sigma_s^2)$.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \epsilon_t^s \tag{2}$$

No assumptions, such as serial correlation, are imposed on the shock terms ϵ_t^d or ϵ_t^s since they have no bearing the policy equations or empirical exercise as will be shown below.

Both commitment and discretionary policy are derived from an optimizing agent framework. The monetary authority faces preferences that are quadratic in nature derived by (3).

$$L_t = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t - \pi_t^*\right)^2 + \frac{\phi}{2} x_t^2 + \frac{\delta}{2} \left(i_t - i_t^*\right)^2 \right] \tag{3}$$

The central bank, which is assumed to act optimally, seeks to minimize societal loss, L_t which is the conditional expectational sum of future losses. Deviations of inflation from a time varying inflation target (π_t^*) are normalized to one. ϕ and δ are the relative aversion parameters to deviations in output from natural output and the interest rate from an implicit target (i_t^*) respectively. Larger values for these parameters imply are stronger aversion (and thus a higher degree of associated loss) to deviations.

$$\pi_t^* = \rho \pi_{t-1}^* + \epsilon_t^\pi, \tag{4}$$

The inflation target is assumed to follow a first-order autoregressive process according to Equation (4). This is a similar formulation to Dossche and Everaert (2005), Leigh (2008) and Scott (2016).

$$i_t^* = \bar{r} + \pi_t^*. \tag{5}$$

The interest rate target is assumed to follow the same underlying dynamics as the inflation target since they are linked according to the Fisher equation. The interest rate target is assumed to follow the inflation target according to (5)[5].

[5]Another specification for the inflation target might also look like $\pi_t^* = \alpha + \rho \pi_{t-1}^* + \epsilon_t^\pi$. Under this specification however α and $\bar{r}$ would not be independently identifiable in the empirical estimation. Thus by assuming that the inflation target follows an AR(1) with no drift, identification of $\bar{r}$ is possible, otherwise the constant in Equation 19 would be a nuisance parameter.

2.1 Optimal commitment policy

Optimal commitment policy suffers from an initial condition problem. For policy makers to commit to a policy in the past there must necessarily be a point of reference where the initial conditions of the economy are specified. To circumvent this problem commitment policy can be defined according to Woodford et al. (1999)'s timeless perspective policy (the policy that the central bank wished it had committed to at some point in the distant past). Using this definition, optimal commitment policy can be found by minimizing L_t, Equation 3, w.r.t $\{\pi_t,\ y_t,\ i_t\}_0^\infty$ subject to (1) and (2). The generalized system of linear first-order necessary conditions (FOC) for this problem are characterized by Equations (6) - (10).

$$
\begin{aligned}
(\pi_t - \pi_t^*) - \lambda_{\pi t} &= 0, \quad t = 1 && (6)\\
\phi x_t + \kappa \lambda_{\pi t} - \lambda_{xt} &= 0, \quad t = 1 && (7)\\
(\pi_t - \pi_t^*) + \lambda_{\pi t-1} + \frac{\sigma}{\beta}\lambda_{xt-1} - \lambda_{\pi t} &= 0, \quad t = 2, 3, \ldots && (8)\\
\phi x_t - \frac{1}{\beta}\lambda_{xt-1} + \kappa \lambda_{\pi t} - \lambda_{xt} &= 0, \quad t = 2, 3, \ldots && (9)\\
\delta\left(i_t - i_t^*\right) - \sigma \lambda_{xt} &= 0, \quad t = 1, 2, 3, \ldots && (10)
\end{aligned}
$$

These FOC's define the time inconsistency problem of the central bank. Commitment policy arises from Equations (8), (9) and (10) while discretionary policy arises from (6), (7) and (10). Using Equations (8) - (10) and the law of iterated expectations, we can define the FOC for optimal commitment interest rate policy as Equation 11.

$$
\begin{aligned}
0 \;=\; & (\pi_t - \pi_t^*) + \frac{\delta}{\sigma\kappa}\left(i_{t-1} - i_{t-1}^*\right) + \frac{\delta}{\sigma\beta\kappa}\left(i_{t-2} - i_{t-2}^*\right) - \frac{\phi}{\kappa}x_{t-1} \\
+ \; & \frac{\delta}{\beta}\left(i_{t-1} - i_{t-1}^*\right) - \frac{\delta}{\sigma\kappa}\left(i_t - i_t^*\right) - \frac{\delta}{\sigma\beta\kappa}\left(i_{t-1} - i_{t-1}^*\right) + \frac{\phi}{\kappa}x_t \qquad (11)
\end{aligned}
$$

(11) reduces after some small algebra to (12).

$$\Delta i_t = \frac{\sigma\kappa}{\delta}\{\pi_t - \pi_t^*\} + \Delta i_t^* + \frac{\sigma\phi}{\delta}\Delta x_t + \frac{\sigma\kappa}{\beta}\{i_{t-1} - i_{t-1}^*\} - \frac{1}{\beta}\{\Delta i_{t-1} - \Delta i_{t-1}^*\} + \epsilon_t^c \tag{12}$$

This interest rate policy rule is similar to Dennis (2010) in form, but it implies more persistence since the inflation and interest rate target are not assumed constant. Commitment policy also implies persistence in the output gap, this is the so called speed limit to monetary policy first discussed by Walsh (2003).

The FOC's contained in (6) - (10) also contain an optimal path for inflation under both discretion and timeless perspective commitment policy. Optimal inflation under a strategy of timeless perspective commitment is given by Equation 8. After some similar algebra Equation 8 yields (13).

$$\pi_t = \pi_t^* - \frac{\phi}{\kappa}\Delta x_t + \frac{\delta}{\sigma\kappa}[\Delta i_t - \Delta i_t^*] + \frac{\delta}{\sigma\beta\kappa}\left[\Delta i_{t-1} - \Delta i_{t-1}^*\right] - \frac{\delta}{\beta}\left(i_{t-1} - i_{t-1}^*\right) \tag{13}$$

This optimal rule for inflation captures much of the same underlying persistence as the interest rate rule in Equation 12.

2.2 Optimal discretionary policy

Discretionary policy does not suffer from the same initial condition problem as commitment policy. Here every period is an initial condition by which policy makers respond. Discretionary policy can then be found by substituting (1) and (2) into the static form of (3) at $t = 1$ which results in (14).

$$\begin{aligned} L_t &= E_t \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{1}{2}\right) \left(\beta E_t \pi_{t+1} + \kappa \left(E_t x_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) + \varepsilon_t^d\right) + \varepsilon_t^s - \pi_t^*\right)^2 \right. \\ &+ \left. \frac{\phi}{2}\left(E_t x_{t+1} - \sigma\left(i_t - E_t\pi_{t+1}\right) + \varepsilon_t^d\right)^2 + \frac{\delta}{2}\left(i_t - i_t^*\right)^2 \right] \end{aligned} \tag{14}$$

Minimizing Equation 14 w.r.t. i_t implies the following FOC[6].

$$-\sigma\left\{\kappa\left[(\pi_t-\pi_t^*)\right]+\phi x_t\right\}+\delta\left(i_t-i_t^*\right)=0 \tag{15}$$

or

$$\Delta i_t=\frac{\sigma\kappa}{\delta}\{\pi_t-\pi_t^*\}+\frac{\sigma\phi}{\delta}x_t+i_t^*-i_{t-1}+\epsilon_t^d \tag{16}$$

Note here that the policy rule under discretion is not characterized in levels but in differences in order to show that optimal discretionary interest rate policy is nested within optimal commitment interest rate policy. The backward looking dynamics are no longer present under a discretionary equilibrium, highlighting once again the dynamic tradeoff over time that the central bank faces.

Optimal inflation under discretion is found similar to the commitment case, but here it is given by (6). After some similar algebra we get

$$\pi_t=\pi_t^*-\frac{\phi}{\kappa}x_t+\frac{\delta}{\sigma\kappa}\{i_t-i_t^*\}+\frac{\delta}{\sigma\beta\kappa}\{i_{t-1}-i_{t-1}^*\} \tag{17}$$

Here also, optimal inflation under discretion implies less persistence than optimal inflation under commitment.

3 Data and method

The empirical estimation of this model will center around the joint estimation of the interest rate policy rule and the unobserved inflation target dynamics. Since the interest rate target dynamics are the same as the inflation target dynamics, this can be iteratively substituted out of Equation

[6] One can also use (6), (7), and (10) to get to (15).

(12) and simplifies to (18)

$$\Delta i_t = \frac{\sigma\kappa}{\beta}\bar{r} + \frac{\sigma\kappa}{\delta}\{\pi_t - \pi_t^*\} + \Delta\pi_t^* + \frac{\sigma\phi}{\delta}\Delta x_t + \frac{\sigma\kappa}{\beta}\{i_{t-1} - \pi_{t-1}^*\} - \frac{1}{\beta}\{\Delta i_{t-1} - \Delta\pi_{t-1}^*\} + \epsilon_t^c \quad (18)$$

A reduced form version of this commitment policy rule is then represented by,

$$\Delta i_t = c\bar{r} + a\{\pi_t - \pi_t^*\} + \Delta\pi_t^* + b\Delta x_t + c\{i_{t-1} - \pi_{t-1}^*\} - d\{\Delta i_{t-1} - \Delta\pi_{t-1}^*\} + \epsilon_t^c \quad (19)$$

where $a = \frac{\sigma\kappa}{\delta}$, $b = \frac{\sigma\phi}{\delta}$, $c = \frac{\sigma\kappa}{\beta}$, and $d = \frac{1}{\beta}$. Here the timeless perspective policy rule is measured in differences. It contains a few familiar components such as an inflationary gap and an output gap, but also contains a lag in the output gap and some additional persistence beyond a simple non-dynamic Taylor (1993) rule.

The discretion policy model can be found similarly. Substituting (5) into (16) produces (20)

$$\Delta i_t = \bar{r} + \frac{\sigma\kappa}{\delta}\{\pi_t - \pi_t^*\} + \frac{\sigma\phi}{\delta}x_t + \pi_t^* - i_{t-1} + \epsilon_t^d \quad (20)$$

which can then be expressed in a reduced form as (21).

$$\Delta i_t = \bar{r} + a\{\pi_t - \pi_t^*\} + bx_t + \pi_t^* - i_{t-1} + \epsilon_t^d \quad (21)$$

The reduced form parameters a and b are defined as in (19). Discretionary policy necessarily implies a greater sensitivity to the ever changing initial conditions for the economy, thus the policy rule imparts less persistence on the interest rate. Additionally, it is worth noting that the discretionary policy rule (both reduced and structural) is nested in the commitment policy rule. Thus discretionary policy is a special case of commitment policy.

3.1 Estimation strategy

Equations (19) and (4) represent the system of processes to be estimated. Together these equations can be cast into state-space form. π_t^* and ϵ_t^π are unobserved components whose evolution through time comprises the state equation. The first difference of the interest rate, Δi_t defines the observation equation which is related to the unobserved state through theory via the coefficients in (19). The log likelihood function for the system is directly observable by

$$ll = \sum_{t=1}^{T} \ln \left[(2\pi)^{-\frac{1}{2}} \det \left(v_{t|t-1} \right)^{-\frac{1}{2}} \exp \left(-0.5 e'_{t|t-1} v_{t|t-1} e_{t|t-1} \right) \right],$$

and is iteratively updated using the Kalman filter recursion[7].

The system may suffer from a potential endogeneity problem given the theoretical underpinnings of the interest rate policy rule defined by (1) and (2). Endogeneity is addressed using the two step procedure outlined Chang-Jin et al. (2010). Second lags of inflation and the output gap are used in order to avoid an errors-in-variables problem. The model is estimated employing Bayesian methods. A random walk Metropolis-Hastings MCMC algorithm is used to calculate the marginal likelihood, the sum of the log likelihood and the log prior likelihood, of the model at every draw and construct the joint posterior distribution of the model parameters.

3.2 Priors

The prior distributions for the model parameters are chosen either according to previous literature or to reflect specific properties inherit in the data. Table 1 summarizes the prior distributions for all the chosen model parameters. The parameters a and b are set according to Rabanal and Rubio-Ramírez (2005). These are chosen so as to leave the distributions relatively wide. Note the empirical equation is estimated in differences rather than the level of interest rate which is the

[7] The reader is referred to (Hamilton, 1994, Ch. 13) for a more thorough exposition of the state-space form and the corresponding Kalman filter recursion.

Table 1: Prior distributions

Parameter	Distribution	0.05 pc.	Mean	0.95 pc.
a	Normal $(1.5, 0.25)$	0.5	1.5	2.5
b	Normal $(0.125, 0.13)$	-0.582	0.125	0.832
c	Normal $(0.75, 0.13)$	0.043	0.75	1.457
d	Normal $(0.2, 0.25)$	-0.8	0.2	1.2
$\bar{r}$	Normal $(-0.191, 0.13)$	-0.0418	-0.0191	0.0036
ρ	Normal $(0.5, 0.25)$	-0.5	0.5	1.5
$\sigma^2_{\epsilon_c}$	Inv. Gamma $(2, 1)$	0.355	1.678	4.744
$\sigma^2_{\epsilon_\pi}$	Inv. Gamma $(1, 0.17)$	0.009	0.1188	0.510

case for Rabanal and Rubio-Ramírez (2005). Because of this it is not expected that the posterior distributions will be as wide particularly for the a coefficient on the contemporaneous inflationary gap since the equation is stationary by design. In order to capture a large array of possible values with which to describe past dynamics, priors for c and d are left relatively wide. This is done because different policy data implies differing degrees of persistence and so as to not impose too tight of a parameter space. The constant term, $\bar{r}$, is centered around the unconditional mean of the change in interest rates for all countries. ρ_π is chosen so as to be agnostic regarding the autoregressive parameter. Its distribution is wide enough to even allow for unstable roots in the inflation target. The variance of the policy rule disturbance, $\sigma^2_{\epsilon_c}$, is set according to Scott and Barari (2017). $\sigma^2_{\epsilon_\pi}$ is based from Dossche and Everaert (2005) but is similar to Kozicki and Tinsley (2005) and Smets and Wouters (2005). Collectively these parameter distributions represent a conjugate-normal prior.

3.3 Data

Data used for this estimation is provided from the Federal Reserve Economic Database (FRED) hosted by the St. Louis Federal Reserve bank[8]. The output gap is constructed from the difference in estimated real GDP and an Hodrick-Prescott filter of the same data as a measure of natural output. This series is then converted so that the output gap is a percent. Inflation for all of the countries is

[8]Data notes indicate that some of the series are originally sourced from OECD database.

calculated from consumer price index estimates[9]. The inflation rates are quarterized. The interest rate data is the policy rate for each country's respective central bank. Each one captures differing degrees of market forces and systematic risks according to the idiosyncrasies of the institution[10]. The sample period for each country depends on the availability of the data and is not perfectly aligned. The quarterly data is as follows; Australia 1969:3 to 2013:1, Canada 1961:1 to 2014:4, Japan 1960:1 to 2014:4, United Kingdom 1959:1 to 2014:4, and United States 1955:2 to 2015:1.

4 Results

The estimation employs a random walk Metropolis-Hastings (MH) Markov chain Monte Carlo (MCMC) algorithm. This is because the conditional likelihood of the candidate draws for this model do not have a closed form solution. But since the conditional likelihood is proportional to the product of the model likelihood and the prior, it is possible to evaluate the posterior numerically. The conditional posterior density can be expressed as

$$\pi\left(\theta|y_{1:n-1}, y_n\right) \propto \pi\left(\theta|y_{1:n-1}\right)\pi\left(y_n|\theta, y_{1:n-1}\right) \propto \prod_{t=1}^{n-1}\pi\left(y_t|\theta\right)\pi\left(\theta\right)\pi\left(y_n|\theta\right),$$

where $\pi\left(y_n|\theta, y_{1:n-1}\right)$ represents the likelihood and $\pi\left(\theta|y_{1:n-1}\right)$ represents the model prior which are updated with new measurements and candidate densities. Each new candidate draw is based in part on the previous draw. This is because the unobserved state evolves over time based in part on its previous values. The marginal likelihood of a candidate draw can be calculated as the sum of the log likelihood of the model at that draw and the log prior (loosely, the joint probability that the draw came from the prior distribution). This candidate marginal likelihood is compared to the

[9] It is worth noting that the Federal Reserve is on record as preferring core PCE inflation. Additionally, for the U.S. the Congressional Budget Office provides estimates of natural output. These however are not available for other countries so they are not used in an effort maintain comparability across countries.

[10] All transformed data, MATLAB routine files, and an unpublished appendix with all model details are available from the author upon request.

old draw and either accepted or rejected according to a random draw from a uniform distribution. If the new candidate is accepted then it becomes the old draw in the next iteration. If the new candidate is rejected then the old draw is maintained in the next iteration. In this fashion the joint posterior distribution is constructed. The algorithm starts with an initial candidate equal to a maximum likelihood estimate (the posterior mode) of the model. Additionally, the Hessian matrix from the maximum likelihood estimate dictates the variance of the candidate draws chosen.

In order for the initial candidate at the posterior mode to not unduly impact the estimation there is a burn-in period. A total of 500,000 repetitions of the above algorithm are calculated and the first 250,000 are burned, discarded. Preliminary diagnostics of the model indicate that the model converges before the 100,000th iteration. Table 2 summarizes the posterior distributions of the commitment policy empirical model for the five countries. Generally speaking the posterior distributions for all the parameters moved away from their priors. This can be interpreted as the priors were wide enough to allow the data to speak for itself. Posterior estimates for a, the coefficient on the inflationary gap, and b, the coefficient on the output gap, indicate that they are important to explaining interest rate dynamics. Except for the estimates of b for Canada and Japan, they are all relatively similar in size as well. There is relatively more variation in c and d for the five countries; this reflects the differing degrees of persistence inherent in the data. The autoregressive parameter on the unobserved inflation target, ρ, is highly persistent and important for all countries. This indicates that policy makers are reluctant to make relatively large changes to the implicit inflation target.

4.1 Implicit inflation target

The use of the Kalman filter for this application means that one can backward solve the recursion for the one-step ahead forecasts of the state, which in this model formulation contains the inflation target. Rather than solve for this at the mean, the Carter and Kohn (1994) algorithm is used to sample from the state in order to capture a degree of probabilistic uncertainty[11]. This algorithm draws from the distribution of the state in time. The unobserved state is distributed according to

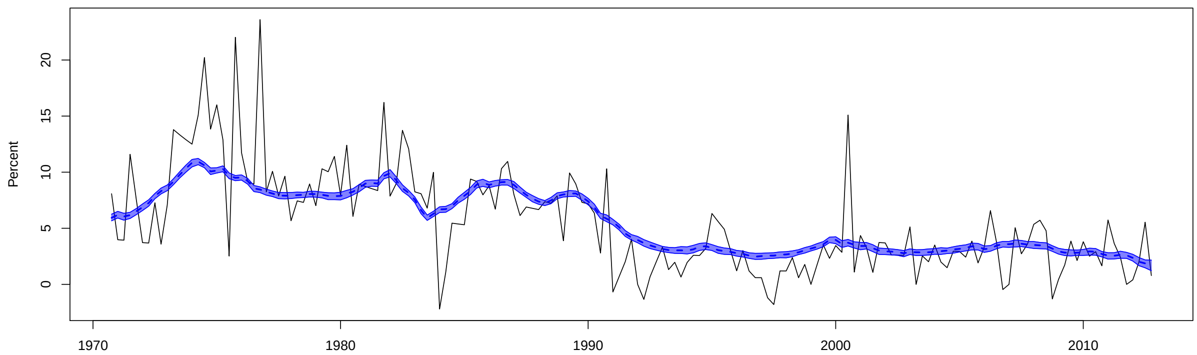
$$\hat{\xi}_{t|t+1} \sim \left(\xi_{t|t,\xi_{t+1}}, v_{t|t,\xi_{t+1}}\right),$$

where $\hat{\xi}_{t|t+1}$ is the one step ahead forecast given updated information at time t that is solved forward to $t+1$. $v_{t|t}$ is the mean squared error (MSE) of the prediction. This means that given a joint posterior distribution of the parameters and a distribution summarizing the marginal likelihood, one can draw repeatedly from the distribution of the state variables over time to construct a probabilistic estimate of the state.

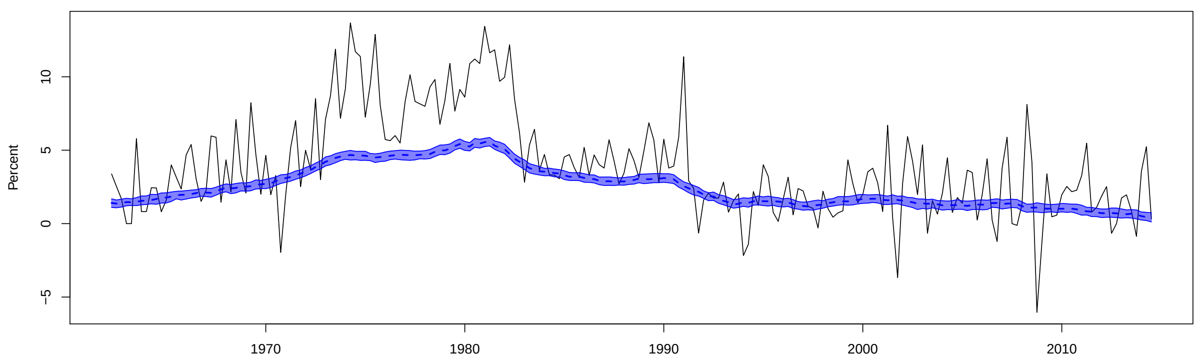
Figure 1 contains the distribution of the state estimates for each country in alphabetical order. Observed inflation is depicted in black while the estimated inflation target band is in blue. The mean of the distribution is denoted by the dotted blue line in the center of the band. The upper bound of the band is the upper 95% and the bottom is the lower 5% of the distribution. Generally speaking the bands tend to follow the smoothed time path of inflation for each country. Some bands are broader than others reflecting a greater degree of uncertainty in the data. For example, the Japan (Figure 1c), U.K. (Figure 1d), and U.S. (Figure 1e) have relatively wide bands for the inflation target while Australia (Figure 1a), and Canada (Figure 1b) have relatively tighter bands. For all countries except Japan, the means of the bands do not reach zero. The lower bound does touch zero for a short while at the end of the series for both U.K. and U.S. This is consistent with previous estimates for the U.S. shown in Scott and Barari (2017). The target at the mean is slightly lower than those reported by Ireland (2007), but there are quite a few notable differences between that study and this estimation. Japan's inflation target distribution is not distinguishable from zero for most of the 2000's, a time period where growth rates in Japan have been close to zero as well. Where observed inflation meets the estimated band can be interpreted as the central bank meeting

[11] Not only uncertainty of the model, but also the parameter space.

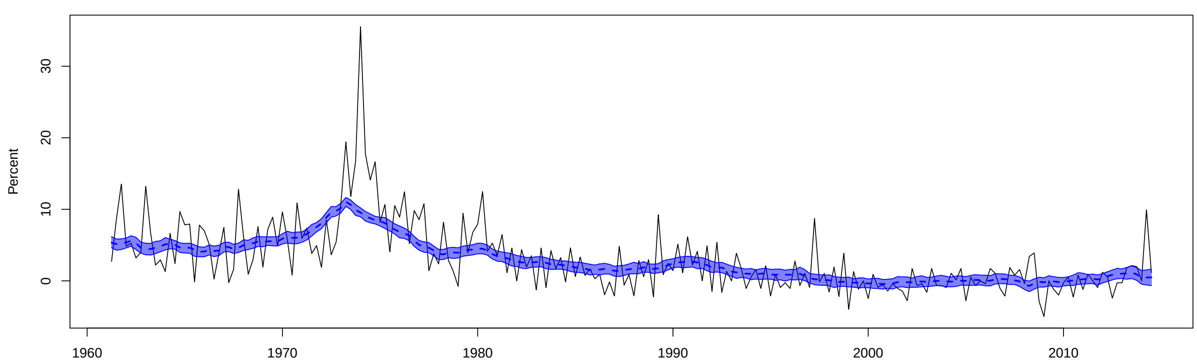
Figure 1: Observed Inflation (black) and Estimated Inflation Target (blue)



(a) Australia

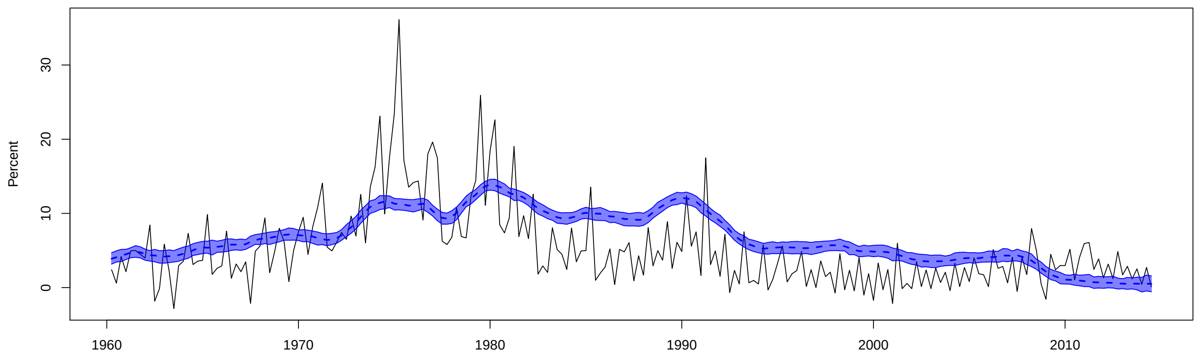


(b) Canada

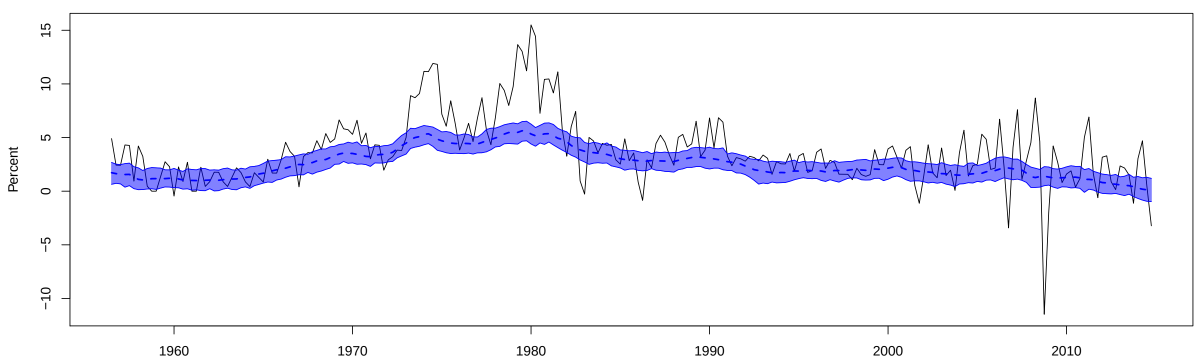


(c) Japan

Figure 1: Observed Inflation (black) and Estimated Inflation Target (blue)



(d) United Kingdom



(e) United States

its inflation target.

4.2 Inflation policy

Using the definitions from (13) along with (5), optimal inflation under timeless perspective policy in a reduced form is given by (22).

$$\pi_t^c = \frac{c}{a}\bar{r} + \pi_t^* - \frac{b}{a}\Delta x_t + \frac{1}{a}\left[\Delta i_t - \Delta\pi_t^*\right] + \frac{d}{a}\left[\Delta i_{t-1} - \Delta\pi_{t-1}^*\right] - \frac{c}{a}\left(i_{t-1} - \pi_{t-1}^*\right) \tag{22}$$

Here the definitions for a, b, c, and d are the same as in (19) and the empirical estimation. Likewise under discretion, substituting (5) into (17) produces in its reduced form

$$\pi_t^d = \bar{r}\frac{1}{a}(d-1) + \pi_t^* - \frac{b}{a}x_t + \frac{1}{a}\{i_t - \pi_t^*\} + \frac{d}{a}\{i_{t-1} - \pi_{t-1}^*\} \tag{23}$$

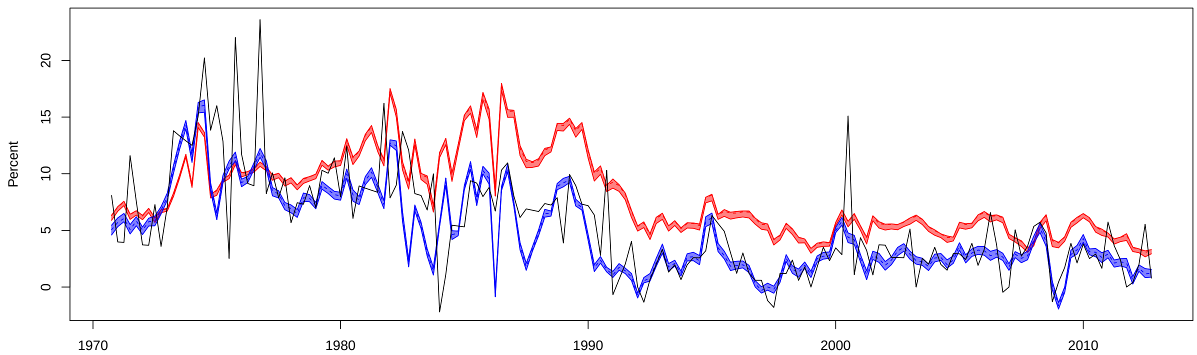
Thus the inflation policy deviation is given $\pi_t^d - \pi_t^c$ or Equation (24).

$$\pi_t^{deviation} = \bar{r}\frac{d-c-1}{a} - \frac{b}{a}x_{t-1} + \frac{1-c}{a}\{i_{t-1} - \pi_{t-1}^*\} - \frac{d}{a}\{i_{t-2} - \pi_{t-2}^*\} \tag{24}$$

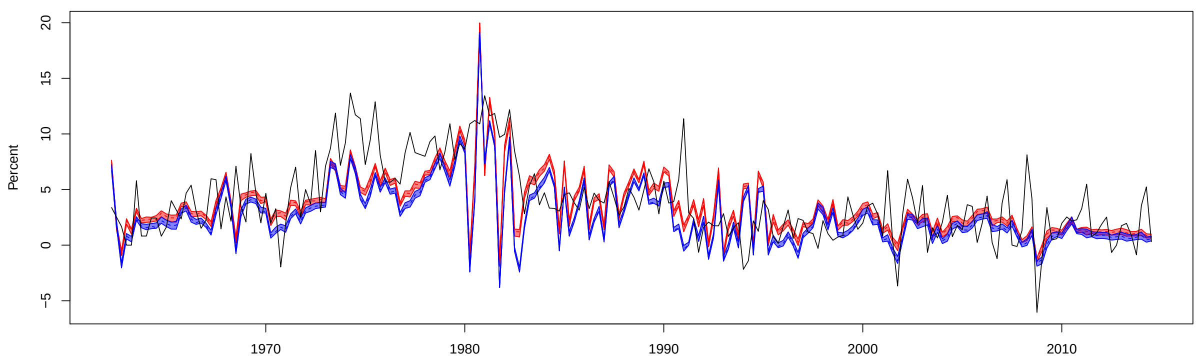
This shows that the inflation policy deviation in this context inherits persistence from the output gap, the interest rate, and the inflation target. It is possible to simulate for the policy deviation using the posterior distributions and the distribution of the inflation target estimates from above. Arguably, it is more intuitive to show the deviations from the inflation simulations alongside observed inflation. Figure 2 is organized similar to Figure 1. Observed inflation is shown in black. The commitment inflation policy bands are in blue while the discretion inflation policy bands are in red. As before, the upper bound of the band is the upper 95% and the bottom is the lower 5% of the distribution. There are a few conclusions that we can draw from these plots. First, there is an observable significant inflation policy deviation for all countries except the U.K. (Figure 2d) and for prolonged periods prior to 2000. Since the colored band indicates statistically equivalent inflation outcomes when the red band is above the blue for a prolonged period then we observe an inflationary policy deviation. When the two colored bands meet the deviation is eliminated. Second for Canada (Figure 2b), U.K. (Figure 2d), and U.S. (Figure 2e) the inflation policy deviation over the entire subsample is relatively small. Australia (Figure 2a) and Japan (Figure 2c) observe a policy deviation for most of their respective sample periods. Third, after 2000 all countries except Australia observe little to no inflation policy deviation. The average inflation deviation over all five countries for the full sample period at the mean of the distributions is 1.6456%.

The first three columns of Table 3 highlight the shrinking of the inflation policy deviation that we see in the posterior estimates. This table is divided into three subsections; full sample period, sample prior to 2000, and the sample post 2000. The first three columns express the average inflation policy deviation as a distribution that can loosely be thought of as a confidence interval around the mean inflation deviation. All three sample periods highlight the degree of variation within the deviation estimates across countries. Australia and Canada have considerably tighter distributions, while Japan, U.K. and U.S. have relatively wider distributions. Prior to 2000 the average inflation policy deviation at all points in the distribution (5%, mean, and 95%) is higher than the full sample average. This implies that post 2000 the average inflation policy deviation is pulling the overall average downward, and this is seen in the bottom portion of the table. The average inflation policy deviation at all points in the distribution is below the full sample average. This implies that the discretion/commitment policy tradeoff is less of a concern before and during the global financial crisis. These results and the evidence shown in Figure 2 highlights that the inflation policy deviation is effectively zero for all countries except for Australia so far in the twenty first century.

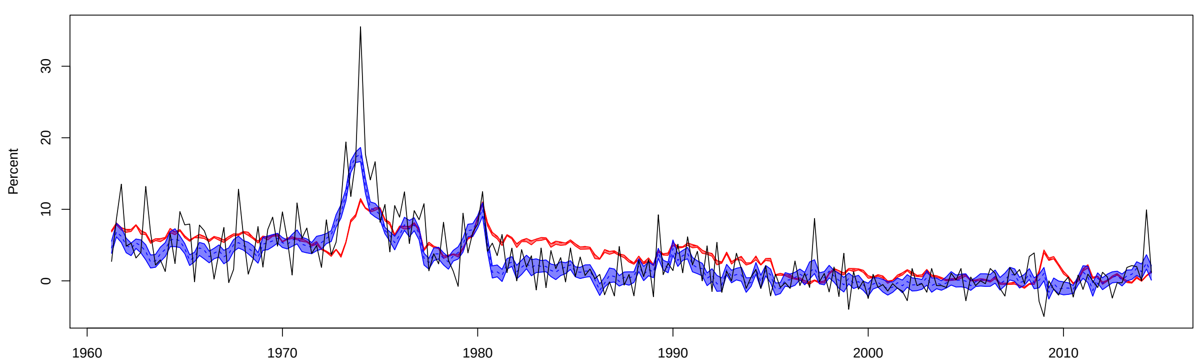
Figure 2: Observed Inflation (black) Along with Discretionary (red) and Commitment (blue) Policy



(a) Australia

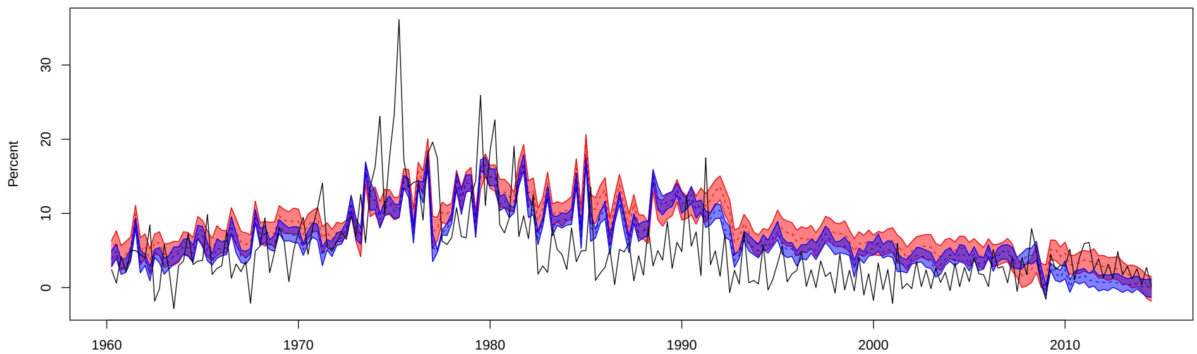


(b) Canada

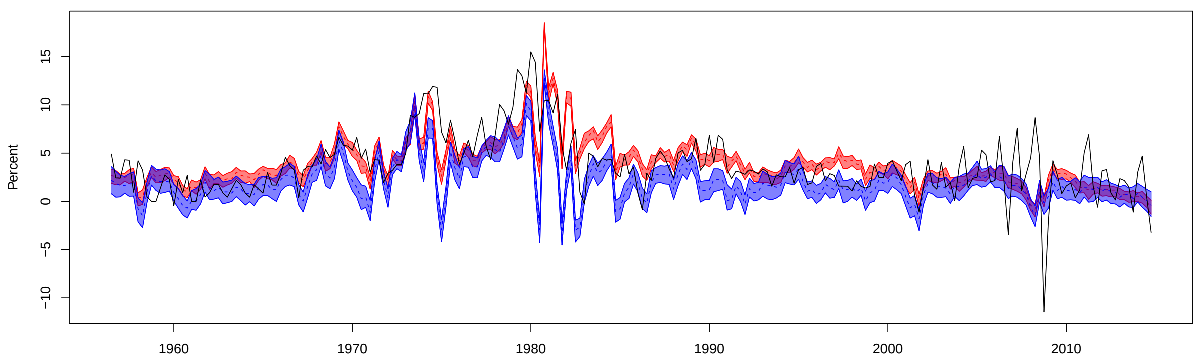


(c) Japan

Figure 2: Observed Inflation (black) Along with Discretionary (red) and Commitment (blue) Policy



(d) United Kingdom



(e) United States

4.3 Interest rate policy

In the same fashion that discretionary inflation is on average higher than commitment inflation, discretionary interest rate policy is on average lower than commitment interest rate policy. Because of the connection between inflation and the interest rate through the Fisher equation, the dynamic inconsistency of monetary policy also induces an interest rate policy deviation that mirrors the inflation policy deviation. This interest rate deviation is observed in Figure 3. As with Figure 1 and Figure 2, each subfigure shows observed data and posterior simulations for each country in alphabetical order. The observed policy rate is in black, the commitment policy band is in blue, and the discretionary policy band is in red. There are a few conclusions that can be drawn from these plots. First, on average the interest rate policy that would occur under a strategy of commitment is statistically higher than its discretionary equivalent for all countries except the U.K. (Figure 3d). This result implies that discretionary policy is more often loose than tight. Policy makers that act in a discretionary fashion are more often dovish than hawkish in terms of interest rate policy. This is consistent with the inflation results from Figure 2. Additionally, it is also consistent with the discretionary asymmetric preferences literature mentioned above. Second, the interest rate policy deviation is either reduced or eliminated for Canada (Figure 3b) and the U.S. (Figure 3e) post 2000. For Japan (Figure 3c), the interest rate policy deviation is eliminated in 1995 while for Canada (Figure 3b) the policy deviation is entirely eliminated until the global financial crisis around 2007. It is reduced at the end of the 1990's for Australia (Figure 3a), but it still remains post 2000. These results largely mirror the inflation results. Third, the average interest rate policy deviation for all five countries over the full sample period at the mean of the distributions is 2.0968%. This reflects the Taylor principle holding for these countries.

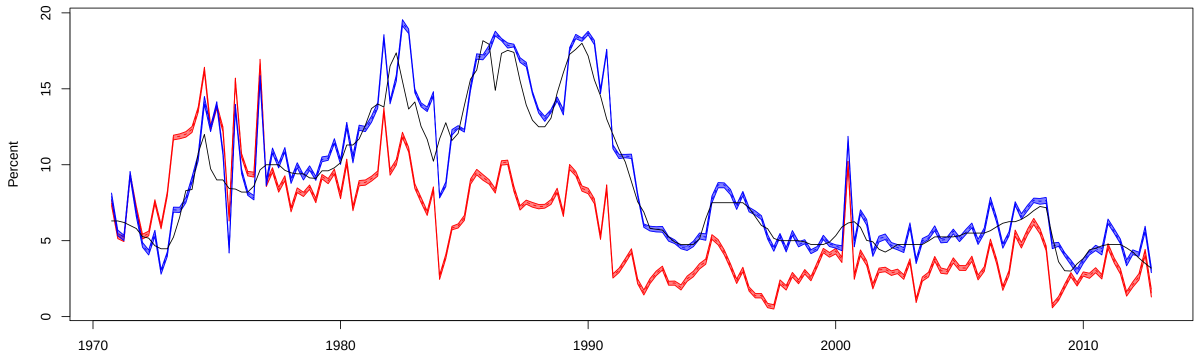
The last three columns of Table 3 show the dynamics of the interest rate policy deviation for the five countries over the full sample period, before 2000 and after 2000. Similar to their inflation counterparts, these columns express can be interpreted as confidence interval for the mean deviation estimate. The relatively tight inflation deviation interval for Australia and Canada is reflected in their respective interest rate deviation interval. Alternately, the wider interval for Japan, U.K., and U.S. are also born out in the interest rate bias interval. Prior to 2000, the average interest rate policy deviation at all points in the distribution (5%, mean, and 95%) is higher than the full sample average. Post 2000, the average decreases pulling the average down for all countries. For all countries except Australia interest rate policy is not substantially different than what commitment policy would prescribe during and coming out of the global financial crisis.

5 Conclusion

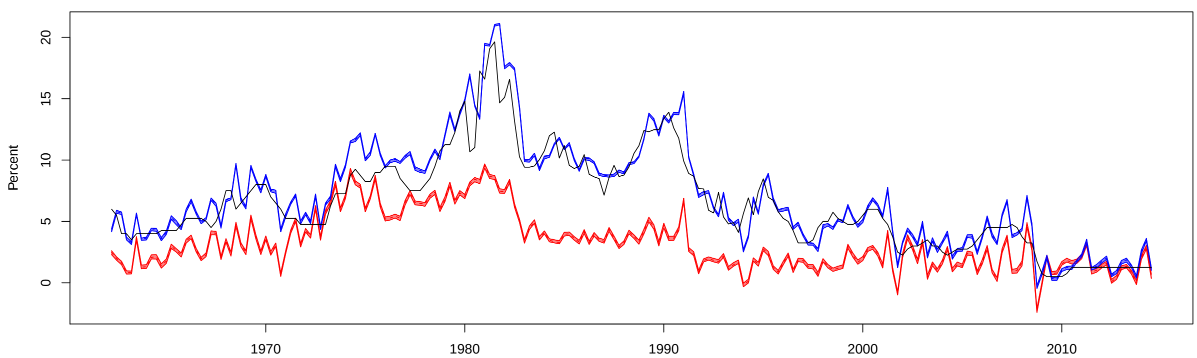
This paper employs a familiar New-Keynesian framework to solve the monetary planner's problem under both commitment (timeless perspective) and discretion. Additionally, the planner's intrinsic policy targets are assumed to vary over time. Optimal discretionary and commitment policy interest rate and inflation rate rules are solved. The model is estimated using Bayesian methods in order to construct a joint posterior distribution of the model parameters. The intrinsic inflation target is also sampled from the posterior distribution of the state. The posteriors are then used to simulate for interest rate and inflation rate policies under the two regimes. The theoretical inflation policy deviation is shown to be a function of lagged values of the output gap, interest rate, and the implicit inflation target.

Results indicate that the inflation policy deviation was historically more important to policy makers than it is today. This deviation has diminished, and in the case of Canada, Japan, U.K., and U.S., is not significantly different from zero after 2000. The time inconsistency of optimal policy also induces an interest rate policy deviation. Commitment policy is shown to be on average higher than discretionary policy. This implies that monetary policy makers tend to be more dovish when they act in a discretionary fashion. The interest rate policy deviation is either reduced or eliminated during and exiting the global financial crisis.

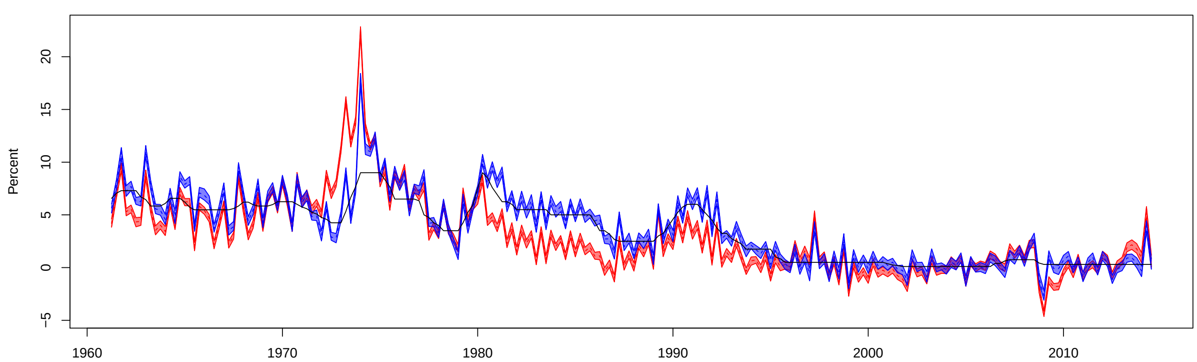
Figure 3: Observed Interest Rate (black) Along with Discretionary (red) and Commitment (blue) Policy



(a) Australia

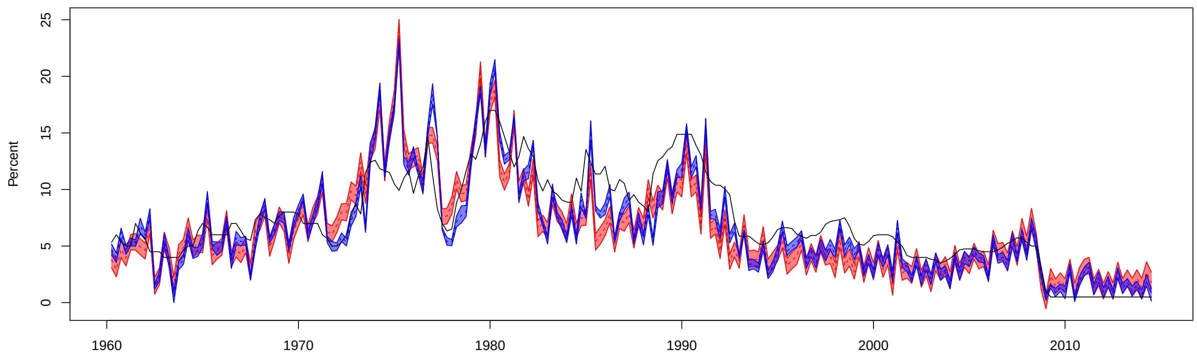


(b) Canada

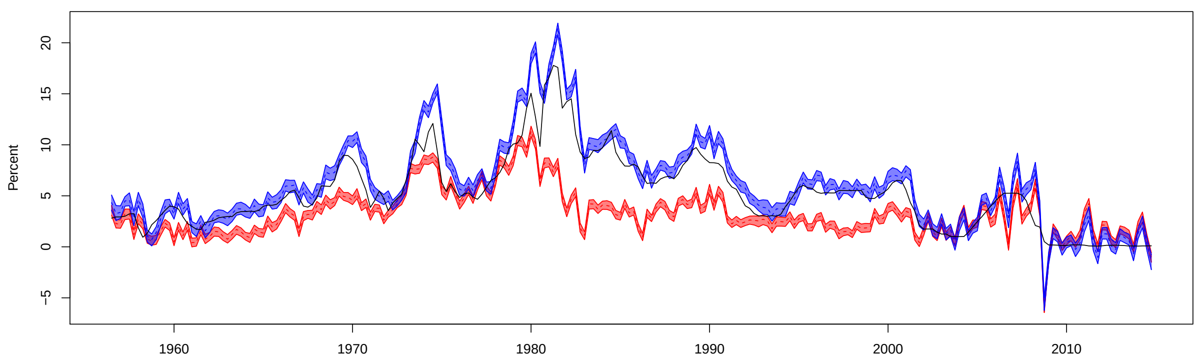


(c) Japan

Figure 3: Observed Interest Rate (black) Along with Discretionary (red) and Commitment (blue) Policy



(d) United Kingdom



(e) United States

Table 2: Posterior distributions - Timeless perspective policy

	Australia - 1969:3 to 2013:1			Canada - 1961:1 to 2014:4			Japan - 1960:1 to 2014:4		
Parameter	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
a	0.5042	0.5048	0.5053	0.4777	0.4796	0.4794	0.4981	0.4985	0.4989
b	0.6576	0.6715	0.6839	0.0989	0.1364	0.1336	0.3156	0.3198	0.3248
c	0.1352	0.1397	0.1438	-0.0342	-0.0230	-0.0239	0.1024	0.1034	0.1042
d	0.5452	0.5673	0.5871	0.2132	0.2168	0.2168	0.8647	0.8954	0.9217
$\bar{r}$	0.2891	0.3136	0.3354	0.0938	0.0943	0.0943	0.1015	0.1021	0.1027
ρ	0.9107	0.9121	0.9133	0.7528	0.7650	0.7641	0.9052	0.9073	0.9091
$\sigma^2_{\epsilon_c}$	0.7137	0.8331	0.9668	6.0400	6.5390	6.5413	0.5020	0.5875	0.6872
$\sigma^2_{\epsilon_\pi}$	0.1532	0.1600	0.1662	0.0212	0.0278	0.0273	0.1015	0.1022	0.1027
Marg. Likelihood	-619.91	-618.48	-617.98	-699.1589	-697.1665	-696.9224	-619.97	-618.52	-618.01

	United Kingdom - 1959:1 to 2014:4			United States - 1955:2 to 2015:1		
Parameter	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
a	0.4992	0.4995	0.4997	0.5460	0.5537	0.5606
b	0.5615	0.5867	0.6121	0.5230	0.5260	0.5295
c	0.0969	0.0980	0.0991	0.0850	0.0867	0.0886
d	-0.0531	0.0777	0.2078	0.3822	0.3933	0.4032
$\bar{r}$	0.1849	0.5241	0.8651	0.1109	0.1127	0.1143
ρ	0.9003	0.9021	0.9039	0.9088	0.9103	0.9116
$\sigma^2_{\epsilon_c}$	2.0012	2.0035	2.0059	0.8095	0.9444	1.0957
$\sigma^2_{\epsilon_\pi}$	0.1001	0.1016	0.1030	0.1247	0.1289	0.1326
Marg. Likelihood	-1005.2474	-1003.8067	-1003.2954	-708.5359	-707.0709	-706.5573

Joint posterior distributions are constructed from the random-walk Metropolis Hastings MCMC algorithm. The policy rule equation is given by (19), $\Delta i_t = c\bar{r} + a\{\pi_t - \pi^*_t\} + \Delta\pi^*_t + b\Delta x_t + c\{i_{t-1} - \pi^*_{t-1}\} - d\{\Delta i_{t-1} - \Delta\pi^*_{t-1}\} + \epsilon^c_t$. The remaining parameters correspond to (4). These equations are estimated jointly.

Table 3: Summary Statistics - Timeless Perspective and Discretionary Policy

Full Sample

	Average inflation policy deviation			Average interest rate policy deviation		
	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
Australia	2.9980	3.2085	3.3003	2.9959	2.9917	2.9947
Canada	0.8088	0.8643	0.9241	3.8193	3.8034	3.7306
Japan	0.3470	1.2119	1.8519	0.6846	0.7482	0.8174
United Kingdom	0.5150	1.1880	1.6649	-0.1900	0.1180	0.4135
United States	1.2459	1.7551	2.1778	2.6781	2.8227	2.9806

Prior 2000

	Average inflation policy deviation			Average interest rate policy deviation		
	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
Australia	3.3673	3.5112	3.6134	3.4294	3.4305	3.4321
Canada	0.9626	0.9966	1.0746	4.6424	4.7253	4.7412
Japan	0.6065	1.3896	2.0530	0.8963	0.9617	1.0371
United Kingdom	0.5385	1.2158	1.7009	-0.0868	0.2063	0.5114
United States	1.6374	2.1412	2.5536	3.4381	3.5848	3.7364

Post 2000

	Average inflation policy deviation			Average interest rate policy deviation		
	0.05 pc.	Mean	0.95 pc.	0.05 pc.	Mean	0.95 pc.
Australia	2.2586	2.4870	2.5446	2.0067	2.0140	2.0143
Canada	0.4503	0.5041	0.5254	1.3969	1.4440	1.4600
Japan	-0.1506	0.5996	1.2854	0.1284	0.1874	0.2403
United Kingdom	0.4700	1.1141	1.5482	-0.4681	-0.1199	0.1498
United States	0.1106	0.6356	1.0879	0.4741	0.6128	0.7888

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